

10. (a)  $1 - \binom{5}{0} \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^5 - \binom{5}{1} \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^4 = 0.539$ . (b)  $\binom{5}{2} \left(\frac{1}{10}\right)^2 \left(\frac{9}{10}\right)^3 = 0.073$ .

26. (a) A four-engine plane is preferable to a two-engine plane if and only if

$$1 - \binom{4}{0} p^0 (1-p)^4 - \binom{4}{1} p (1-p)^3 > 1 - \binom{2}{0} p^0 (1-p)^2.$$

This inequality gives  $p > 2/3$ . Hence a four-engine plane is preferable if and only if  $p > 2/3$ . If  $p = 2/3$ , it makes no difference.

(b) A five-engine plane is preferable to a three-engine plane if and only if

$$\binom{5}{5} p^5 (1-p)^0 + \binom{5}{4} p^4 (1-p) + \binom{5}{3} p^3 (1-p)^2 > \binom{3}{2} p^2 (1-p) + p^3.$$

Simplifying this inequality, we get  $3(p-1)^2(2p-1) \geq 0$  which implies that a five-engine plane is preferable if and only if  $2p-1 \geq 0$ . That is, for  $p > 1/2$ , a five-engine plane is preferable; for  $p < 1/2$ , a three-engine plane is preferable; for  $p = 1/2$  it makes no difference.

7.  $P(X=1) = P(X=3)$  implies that  $e^{-\lambda} \lambda = \frac{e^{-\lambda} \lambda^3}{3!}$  from which we get  $\lambda = \sqrt{6}$ . The answer is  $\frac{e^{-\sqrt{6}} (\sqrt{6})^5}{5!} = 0.063$ .

14. Choose one month as the unit of time. Then  $\lambda = 5$  and the probability of no crimes during any given month of a year is  $P(N(1) = 0) = e^{-5} = 0.0067$ . Hence the desired probability is

$$\binom{12}{2} (0.0067)^2 (1 - 0.0067)^{10} = 0.0028.$$

8. The probability that at least  $n$  light bulbs are required is equal to the probability that the first  $n-1$  light bulbs are all defective. So the answer is  $p^{n-1}$ .

15.  $\sum_{i=3}^5 \frac{\binom{18}{i} \binom{10}{5-i}}{\binom{28}{5}} = 0.772$ .

18.  $1 - \sum_{i=0}^2 \binom{20}{i} (0.06)^i (0.94)^{20-i} = 0.115$ .

Let  $X$  be the number of requests for reservations at the end of the second day. It is reasonable to assume that  $X$  is Poisson with parameter  $3 \times 3 \times 2 = 18$ . Hence the desired probability is

$$P(X \geq 24) = 1 - \sum_{i=0}^{23} P(X=i) = 1 - \sum_{i=0}^{23} \frac{e^{-18} (18)^i}{i!} = 1 - 0.89889 = 0.10111.$$

22. (a)  $\left(\frac{w}{w+b}\right)^{n-1} \left(\frac{b}{w+b}\right)$ . (b)  $\left(\frac{w}{w+b}\right)^{n-1}$ .