

Using Technology to Teach Linear Algebra

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Computing at Stevens Institute of Technology

Stevens Institute of Technology is a relatively small technological university in which about 80% of its undergraduates are engineering majors. It has undergraduate and graduate programs in all branches of engineering, science, and mathematics.

Stevens has a long history of incorporating the use of computers into the educational experiences of its students. Indeed, in 1983 it was the first school in the country to require a group of undergraduates to purchase a PC and use these machines throughout their academic careers. This naturally led to a requirement that all undergraduates have a PC. However, this fall the PC requirement is being replaced with a requirement that all students have a laptop. The goal is that students will use their laptops as part of the teaching/learning experience both within laptop classrooms that contain network and power connections and outside of the classroom while studying and completing assignments.

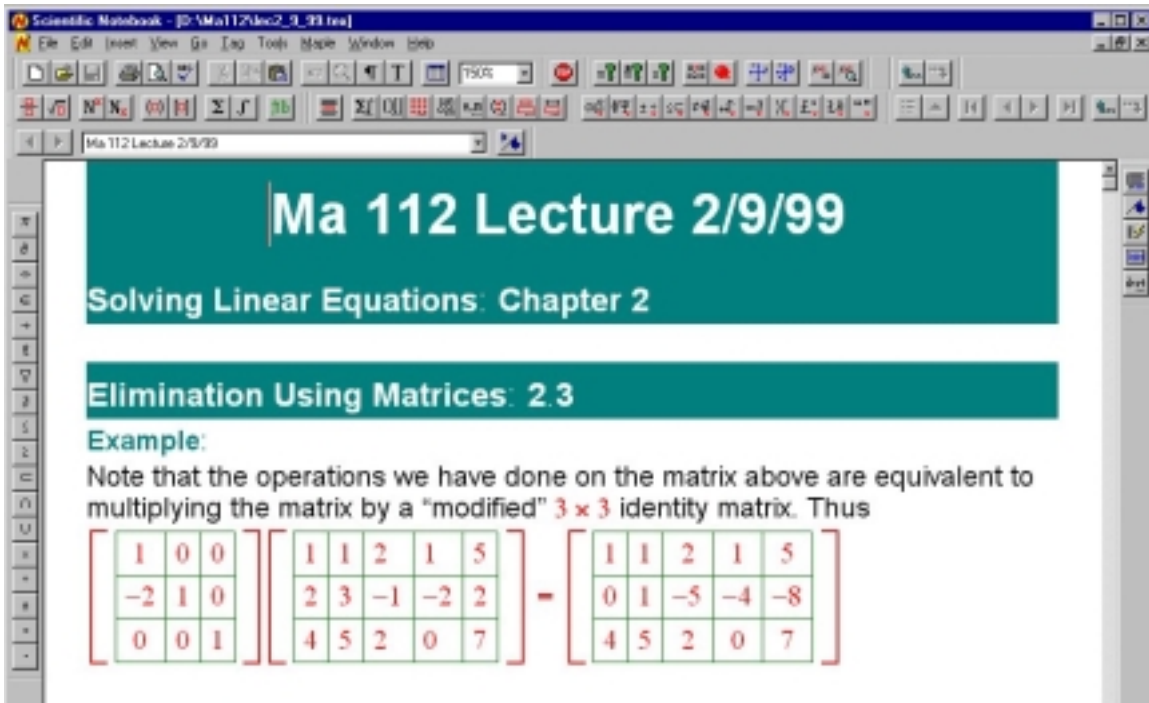
To prepare for the switch from PCs to laptops, Stevens renovated one classroom and purchased about 45 Dell Latitudes this past year. Thirty of these machines were kept in this laptop classroom so that faculty could experiment with teaching in a new environment. The author used this classroom to teach an introductory one-semester course in linear algebra.

Scientific Notebook

The author has used Scientific Notebook (SNB) for several years in the second year mathematics sequence taken by almost all undergraduate students. These two courses cover such topics as ordinary differential equations, Fourier series, partial differential equations, multiple integration, matrix methods, surface integrals, and the theorems of Green, Gauss and Stokes. SNB is a word processing package that incorporates a Maple kernel. This Maple kernel allows one to perform a wide variety of mathematical operations. A particularly nice feature of SNB is that one can use the Maple kernel without typing any code. One simply types the mathematical expression and then clicks

on the appropriate operation to be performed. SNB then performs the operation. Thus the “front end” of SNB is extremely user friendly, and it has been our experience that students are able to use SNB almost from the first introduction with little or no difficulty. The program contains extensive Help files that further foster ease of use.

Information about SNB is available at <http://www.mackichan.com/>. A typical SNB screen is shown below.



In these courses use of SNB was necessarily limited, since students did not have use of computers in the classroom. They were required to use SNB to complete three projects each semester. They were also encouraged to use SNB as a tool for checking their work. For example, when topics dealing with multiple integration were presented, the concentration was on setting up the integrals. The evaluation of these integrals was to be done with SNB. A complete set of notes for the material covered in these courses as well as an extensive database of old exams, all in SNB format, were made available on the Web. Thus while SNB was an important component of these courses, its use was not integral to presenting and learning the material presented in the course. Nonetheless, surveys have indicated that most students have found the use of SNB very helpful.

Structure of the linear algebra course

Given the above it was decided to use SNB as an integral component of the linear algebra course. This meant preparation in SNB of a complete set of notes for the material covered. These notes were used as the basis of each lecture. Whenever possible, the notes incorporated in-class exercises that the students worked on during the lectures. Most of

these in-class exercises required using the Maple kernel in SNB. Thus each class meeting was a combination of lecture and hands-on doing. When the instructor was lecturing, laptops were required to be closed, whereas when the students were working on the exercises, the laptops were open. In this way the class moved seamlessly between the two environments of a “standard” classroom and a computer laboratory.

Homework was assigned regularly and had to be done using SNB. **No written work was accepted. In this spirit the instructor taught a mathematics course in which he did not write one single word on the board the entire semester.**

Use of the Web

Before each class the notes for that class were posted on the instructor’s Web site. A nice thing about SNB is that one can put a tex file on the Web, and then this file can be downloaded by simply clicking on it once the Web browser has been configured to open tex files in SNB. Thus each lecture began with the instructor downloading the lecture from the Web to his laptop and then projecting it for the class to see.

All homework assignments were also posted there. In addition, solutions to homework problems and exams were put on the Web site. A good portion of the quizzes and exams were also given over the Web. (See below.) Thus the Web served as a crucial resource for the course. To see what was actually done go to <http://attila.stevens-tech/edu/~llevine/> and click on the Ma 112 Button.

Exams and Quizzes

Given that use of SNB was an integral part of the course, many of the short quizzes and portions of exams given in the course were designed so that the students had to use SNB to solve the problems. These quizzes and exams had to be downloaded by the students, worked on, and then uploaded to a site. All of the major exams and the final were in two parts. On one part the student worked with the classical paper and pencil, whereas on the other part s/he had to use the Maple kernel in SNB to solve the problems. Having a networked laptop classroom made this approach possible.

Classroom set-up

For security reasons the laptops were kept in a safe in the classroom when not being used. They were set up before the students arrived and then returned to the safe at the conclusion of class. A projector was also kept in the safe and set up before each class. All of this was the responsibility of the teaching assistant who worked with the instructor. The pictures below show the classroom from the front and the rear.



The linear algebra course.

This one semester linear algebra course covered vectors, systems of linear equations, matrices, row reduction, echelon forms, LU-factorization, vector spaces and subspaces, rank, linear independence, basis, dimension, null space, determinants, Cramer's Rule, and eigenvalues and eigenvectors. There was a textbook assigned, but it was used primarily for its homework problems, since the students had access to a complete set of notes dealing with the course material. The majority of students enrolled were second and third year computer science majors.

Engaging the student in the teaching/learning experience

One of the goals of a good educational experience is to engage the students in the teaching/learning experience. Active learning is certainly more desirable than passive learning for a wide range of reasons. However, within the classical lecture format this instructor has found it most difficult to get students actively involved in what is going on in class. A number of times this instructor has had the feeling that students come to class with the idea that, "I was good enough to come to class, now it is the professor's job to pour the material into my head." They seem to shun class participation especially when it comes to mathematics.

Given the approach taken in this linear algebra course, namely prepared lecture notes interspersed with exercises that students were required to work on in class, active student involvement became the rule rather than the exception. Usually the in-class exercises that the students worked on were homework problems that had to be turned in and that were graded. This further encouraged students' active learning in class. In this instructor's experience of over three decades no other format has led to as much "active" learning.

Advantages of this approach

Integrating the use of computer technology into this one semester linear algebra course in the manner described above had the following advantages:

- Students learned to use sophisticated mathematical software as a tool in mastering mathematics.
- Mastery of linear algebra entails being able to perform many arithmetic operations. Students often make mistakes while performing these operations. Using software judiciously avoids this problem and allows the student to concentrate on the underlying concepts.

- As mentioned above, a laptop classroom is actually two classrooms in one: with the covers of the laptops up it is a computer laboratory, whereas with the laptop covers down one has a “standard” classroom. This allows the instructor to seamlessly switch between these environments and exploit the one that is appropriate at a given point during the class meeting.
- The active learning described above is certainly a major advantage of this approach.

Problems encountered

- The time investment required preparing for and teaching such a course is considerable. Preparing a complete set of notes for the course that utilized SNB as a crucial component of the teaching and learning experience was non- simple.
- Preparation of exams that incorporated the meaningful use of SNB required considerably more thought and time than preparation of “standard” exams. The instructor found it challenging to come up with exam problems that both tested the students understanding of the material and did this in a manner that exploited the use of the software in an appropriate manner.
- Some students found it difficult to deal with a mathematics course in which writing was not a key component. Students studying mathematics are accustomed to utilizing writing as part of their learning process. A mathematics course in which one does not have to write very much requires getting used to a new learning environment. Indeed, it took almost all students some time to get used to using software as a key component in studying the material.
- There were a number of technical difficulties encountered vis a vis the hardware. These included network problems, power problems and server problems. However, the instructor was fortunate to have his own dedicated server to use for the course, so that when necessary adjustments could be quickly made. The fact that he also had personnel assisting him who could deal with technical problems expeditiously was of great help.
- As explained above, many of the short quizzes and all of the exams had a computer component. This component was downloaded from a server, worked on by the students and then uploaded to the server upon conclusion. Thus security was very much an issue. The server had to be configured so that a student could not upload a new version of his or her solutions to an exam some time after class was over. In a few instances students did something to their files when they uploaded them which destroyed the formatting of the tex files. To restore this formatting turned out to be nontrivial. (Keep in mind that the only record of this part of a student’s work is the uploaded file, since these files were not saved permanently to the laptop. Thus, if a solution file were lost, the student cannot be given any credit for his/her work.)

Applicability to other mathematics courses

The approach taken with this linear algebra course can certainly be used to teach courses in calculus, differential equations and probability and statistics. Indeed it is this author's opinion that using such an approach in these courses would enhance the learning experience for students.

However, this approach does have limitations. The linear algebra course had 28 students in it. In addition to the instructor there was also a teaching assistant present during all classes. He was helpful in assisting students who ran into problems while working on the laptops. Thus two people were necessary to handle a class of 28. The class size could have been increased to 35 or perhaps even 40 without foreseeable problems. However, forty students is probably the upper limit for such a course. The point is that this approach will not work with a lecture class of say eighty or a hundred.

With this in mind the laptop will not be used in large lecture sections of calculus this fall at Stevens. Instead, the plan is to use the laptop in recitations of at most 30 students in a manner similar to that employed in the linear algebra course.

Student reaction

Students were asked to complete surveys on the web a couple of times during the semester. The vast majority of students liked the course, but there were a few who were very unhappy with the approach. Approximately 90% of the students indicated that they would be willing to take another course that was given in this format.

Perhaps the overall conclusion to be drawn is that while some students will always be unhappy with a course that employs the extensive use of technology, the vast majority will benefit greatly from courses that judiciously employ technology as part of the teaching/learning experience. The challenge is for faculty to find meaningful ways to incorporate this technology into their courses. This is far from a trivial challenge.

