Ma 221 Homework Solutions Spring 2009 Due January 15/16, 2009

1.2 p.14 #1, 3, 5, 6, 7, 9, 8

1.

(a) Show that $\Phi(x) = 2x^3$ is an explicit solution to $x\frac{dy}{dx} = 3y$ on the interval $(-\infty,\infty)$.

Differentiating $\Phi(x)$ gives:

$\Phi'(x) = 6x^2$

Substituting Φ and Φ' for y and y':

$x\frac{dy}{dx} = 3y$

$xy' = 3y$

$x(6x^2) = 3(2x^3)$

$6x^2 = 6x^2$

This identity is true on $(-\infty,\infty)$ and therefore $\Phi(x)$ is an explicit solution on $(-\infty,\infty)$.

(b) Show that $\Phi(x) = e^x - x$ is an explicit solution to $\frac{dy}{dx} + y^2 = e^{2x} + (1-2x)e^x + x^2 - 1$ on the interval $(-\infty,\infty)$.

Differentiating $\Phi(x)$ gives:

$\frac{d\Phi}{dx} = \frac{d}{dx}(e^x - x) = e^x - 1$

Substituting Φ and Φ' for y and y':

$\frac{d\Phi}{dx} + \Phi(x)^2 = (e^x-1) + (e^x - x)^2 = (e^x - 1) + (e^{2x} - 2xe^x + x^2) = e^{2x} + (1-2x)e^x + x^2 - 1$

$e^{2x} + (1-2x)e^x + x^2 - 1 = e^{2x} + (1-2x)e^x + x^2 - 1$

Since both sides of the equation are equal, $\Phi(x)$ is an explicit solution on $(-\infty,\infty)$.

(c) Show that $\Phi(x) = x^2 - x^{-1}$ is an explicit solution to $x^2\frac{d^2y}{dx^2} = 2y$ on the interval $(-\infty,\infty)$.

Differentiating $\Phi(x)$ twice gives:

$\frac{d\Phi}{dx} = \frac{d}{dx}(x^2 - x^{-1}) = 2x - (-1)x^{-2} = 2x + x^{-2}$

$\frac{d^2\Phi}{dx^2} = \frac{d}{dx}(\frac{d\Phi}{dx}) = \frac{d}{dx}(2x + x^{-2}) = 2 + (-2)x^{-3} = 2(1 - x^{-3})$

Therefore

$x^2\frac{d^2\Phi}{dx^2} = x^2.2(1 - x^{-3}) = 2(x^2 - x^{-1}) = 2\Phi(x)$

Both side of the equation are equal therefore, $\Phi(x)$ is an explicit solution to the differential equation $x^2y'' = 2y$ on any interval that does not contain the point $x = 0$, in particular, on $(0,\infty)$.

3. Determine whether the function $y = \sin x + x^2$ is a solution to the differential equation $\frac{d^2y}{dx^2} + y = x^2 + 2$.

Since $y = \sin x + x^2$, we have $y' = \cos x + 2x$ and $y'' = -\sin x + 2$. These functions are defined on $(-\infty, \infty)$ Substituting these expressions into the differential equation $y'' + y = x^2 + 2$ gives $y'' + y = -\sin x + 2 + \sin x + x^2 = 2 + x^2 = x^2 + 2$ for all x in $(-\infty, \infty)$.

Therefore, $y = \sin x + x^2$ is a solution to the differential equation on the interval $(-\infty, \infty)$.

5. Determine whether the given funtion is a solution to the given differential equation:

$x = \cos 2t \qquad\qquad \frac{dx}{dt} + tx = \sin 2t$

Differentiating $x(t) = \cos 2t$, we get:

$\frac{dx}{dt} = \frac{d}{dt}(\cos 2t) = (-\sin 2t)(2) = -2\sin 2t$

$\frac{dx}{dt} + tx = -2\sin 2t + t\cos 2t$

which is not equal to $\sin 2t$. Therefore, $x(t)$ is not a solution to the given differential equation.

6. Determine whether the given funtion is a solution to the given differential equation:

$\theta = 2e^{3t} - e^{2t} \qquad\qquad \frac{d^2\theta}{dt^2} - \theta\frac{d\theta}{dt} + 3\theta = -2e^{2t}$

Differentiating $\theta = 2e^{3t} - e^{2t}$, we get

$\theta' = 6e^{3t} - 2e^{2t} \qquad \theta'' = 18e^{3t} - 4e^{2t}$

$\frac{d^2\theta}{dt^2} - \theta\frac{d\theta}{dt} + 3\theta = 18e^{3t} - 4e^{2t} - \theta(6e^{3t} - 2e^{2t}) + 3(2e^{3t} - e^{2t}) =$
$24e^{3t} - 7e^{2t} + 2\theta e^{2t} - 6\theta e^{3t} \neq -2e^{2t}$.

Therefore, θ is not a solution to the given differential equation.

7. Determine whether the given funtion is a solution to the given differential equation:

$y = e^{2x} - 3e^{-x} \qquad\qquad \frac{d^2y}{dy^2} - \frac{dy}{dx} - 2y = 0$

Differentiating $y = e^{2x} - 3e^{-x}$, we get

$y' = 2e^{2x} + 3e^{-x}; \qquad y'' = 4e^{2x} - 3e^{-x}$

$\frac{d^2y}{dy^2} - \frac{dy}{dx} - 2y = 4e^{2x} - 3e^{-x} - 2e^{2x} - 3e^{-x} - 2(e^{2x} - 3e^{-x}) = 0.$

Therefore, $y = e^{2x} - 3e^{-x}$is an explicit solution to the given differential equation.

8. Determine whether the given funtion is a solution to the given differential equation:

$y = 3\sin 2x + e^{-x} \qquad\qquad y'' + 4y = 5e^{-x}$

Differentiating $y(x) = 3\sin 2x + e^{-x}$ twice, we get:

$\frac{dy}{dx} = \frac{d}{dx}(3\sin 2x + e^{-x}) = 6\cos 2x - e^{-x}$

$\frac{d^2y}{dx^2} = \frac{d}{dx}(6\cos 2x - e^{-x}) = -12\sin 2x + e^{-x}$

$$y'' + 4y = -12\sin 2x + e^{-x} + 4(3\sin 2x + e^{-x}) = -12\sin 2x + e^{-x} + 12\sin 2x + 4e^{-x} = 5e^{-x}$$

which is equal to $5e^{-x}$. Therefore, $y(t)$ is a solution to the given differential equation.