Ma 221 Homework Solutions Spring 2009 Due January 20, 2009

1.2 p.14 # 4, 9, 11, 15, 20b, 21b, 22a,b
(The underline denotes the problems that are handed in as part of the HW)

In problem 4, determine whether the given function is a solution to the given differential equation.

4.) $x = 3\cos t - 5\sin t, \qquad x'' + x = 0$
$x'' = -3\cos t + 5\sin t$
Now substitute the given function in order to see if it is a solution.
$-3\cos t + 5\sin t + 3\cos t - 5\sin t = 0$
Simplify to get:
$0 = 0$
And, therefore the given function is a solution to the given differential equation.

In problems 9 and 11, determine whether the given relation is an implicit solution to the given differential equation.

9.) $x^2 + y^2 = 6, \qquad \frac{dy}{dx} = \frac{x}{y}$
Implicitly differentiate to get:
$2x + 2yy' = 0$
Solve to get:
$y' = -\frac{x}{y}$
Therefore, $\frac{x}{y}$ is not a solution.

11.) $e^{xy} + y = x - 1, \qquad \frac{dy}{dx} = (e^{-xy} - y)/(e^{-xy} + x)$
Differentiate implicitly to get:
$\frac{d}{dx}(e^{xy} + y) = \frac{d}{dx}(x - 1)$
$e^{xy}\frac{d}{dx}(xy) + \frac{dy}{dx} = 1$
$e^{xy}(y + x\frac{dy}{dx}) + \frac{dy}{dx} = 1$
$\frac{dy}{dx}(e^{xy}x + 1) = 1 - e^{xy}y$
$xe^{xy}y' + ye^{xy} + y' = 1$
Now, solve for y':
$y' = (1 - ye^{xy})/(xe^{xy} + 1)$
Multiply the expression by e^{xy}/e^{xy} :
$y' = (e^{-xy} - y)/(e^{-xy} + x)$
Thus, it is a solution.

15.) Show that $\varphi(x) = Ce^{3x} + 1$ is a solution to $\frac{dy}{dx} - 3y = -3$ for any choice of the constant C.
Differentiate $\varphi(x)$ to get:

$\varphi'(x) = 3Ce^{3x}$
Now substitute φ with y and φ' with y' so that:
$y' - 3y = 3Ce^{3x} - 3(Ce^{3x} + 1) = -3$
Simplify to get:
$-1 = -1$
Which is true for any constant C.

For problem 20, determine for which values of m the function $\varphi(x) = e^{mx}$ is a solution to the given equation.

20.)
b.) $\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$
$\varphi(x) = e^{mx}$
$\varphi'(x) = me^{mx}$
$\varphi''(x) = m^2e^{mx}$
$\varphi'''(x) = m^3e^{mx}$
Now, substitute into the DE to get:
$m^3e^{mx} + 3(m^2e^{mx}) + 2(me^{mx}) = 0$
$m^3 + 3m^2 + 2m = 0$
$m(m^2 + 3m + 2) = 0$
$m(m+2)(m+1) = 0$
$m = 0, -1, -2$

22.) Verifty that the function $\varphi(x) = c_1e^x + c_2e^{-2x}$ is a solution to the linear equation
$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$
for any choice of c_1 and c_2 so that each of the following initial conditions is satisfied:
(a.) $y(0) = 2, \quad y'(0) = 1$
(b.) $y(1) = 1, \quad y'(1) = 0$

$\varphi(x) = c_1e^x + c_2e^{-2x}$
$\varphi'(x) = c_1e^x - 2c_2e^{-2x}$
$\varphi''(x) = c_1e^x + 4c_2e^{-2x}$
Substituting into the DE we get:
$c_1e^x + 4c_2e^{-2x} + c_1e^x - 2c_2e^{-2x} - 2(c_1e^x + c_2e^{-2x}) = 0$
Which simplifies to 0=0 and is a solution.
(a.) Plugging in the initial conditions we get:
$\varphi(0) = c_1 + c_2 = 2$
$\varphi'(0) = c_1 - 2c_2 = 1$
Solving the system, we get $c_1 = \frac{5}{3}$ and $c_2 = \frac{1}{3}$.
(b.) Plugging in the initial conditions we get:
$\varphi(1) = c_1e + c_2e^{-2} = 1$
$\varphi'(1) = c_1e - 2c_2e^{-2} = 0$
Solving the system, we get $c_1 = \frac{2}{3e}$ and $c_2 = \frac{1}{3e^{-2}}$.