MA 221 Homework Solutions
Due date: Tuesday, February 10, 2009

4.4 pg. 186 # 5, 11, 12, 13, 15, 16, 21, 22
(Underlined Problems are to be handed in)

In problems 5 determine whether the method or not the method of undetermined coefficients can be applied to find a particular soultion.

5) $y''(\theta) + 3y'(\theta) - y(\theta) = \sec\theta$

Since $\sec\theta = \frac{1}{\cos\theta}$, the method of undetermined coefficients is not applicable.

In problems 11, 12, 13, 15, 16, 21 and 22 find a particular solution to the differential equation.

11.) $2z'' + z = 9e^{2t}$
$z_p = Ae^{2t}$
$z_p' = 2Ae^{2t}$
$z_p'' = 4Ae^{2t}$
$2(4Ae^{2t}) + Ae^{2t} = 9e^{2t}$
$9Ae^{2t} = 9e^{2t}$
$A = 1$
$z_p = e^{2t}$
Or $p(\lambda) = 2\lambda^2 + 1$ $\quad p(2) = 9 \neq 0$ so $z_p = \frac{9e^{2t}}{9} = e^{2t}$

12.) $2x' + x = 3t^2$
$x_p = (At^2 + Bt + C)$
$x_p' = 2At + B$
$2(2At + B) + At^2 + Bt + C = 3t^2$
$At^2 + (4A + B)t + (2B + C) = 3t^2$
$A = 3; 4A + B = 0; 2B + C = 0$
$A = 3; B = -12, C = 24$
$x_p = 3t^2 - 12t + 24$

13.) $y'' - y' + 9y = 3\sin 3t$
$y_p = A\sin 3t + B\cos 3t$
$y_p' = 3A\cos 3t - 3B\sin 3t$
$y_p'' = -9A\sin 3t - 9B\cos 3t$
$-9A\sin 3t - 9B\cos 3t - 3A\cos 3t + 3B\sin 3t + 9A\sin 3t + 9B\cos 3t = 3\sin 3t$
$3B\sin 3t - 3A\cos 3t = 3\sin 3t$
$A = 0, B = 1$
$y_p = \cos 3t$
Or, consider a companion equation $v'' - v' + 9v = 3\cos 3t$.
Multiply the original DE by i and add it to the companion equation, and let $w = iy + v$.

Then we have

$$w'' - w' + 9w = 3(i\sin 3t + \cos 3t) = 3e^{3it}$$

$p(\lambda) = \lambda^2 - \lambda + 9$ so $p(3i) = -9 - 3i + 9 = 3i \neq 0.$

Therefore

$$w_p = \frac{3e^{3it}}{3i} = -i(\cos 3t + i\sin 3t)$$

Now

$$y_p = \operatorname{Im} w_p = \cos 3t$$

15.) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = xe^x$

$y_p = (Ax + B)e^x$

$y_p' = (Ax + B + A)e^x$

$y_p'' = (Ax + B + 2A)e^x$

$(Ax + B + 2A)e^x - 5(Ax + B + A)e^x + 6(Ax + B)e^x = xe^x$

$(2Ax - 3A + 2B)e^x = xe^x$

$2A = 1 => A = 1/2$

$-3A + 2B = 0 => B = 3/4$

$y_p = (\frac{x}{2} + \frac{3}{4})e^x$

16) $\theta''(t) - \theta(t) = t\sin t$

$r^2 - 1 = 0 \quad \Rightarrow \quad r = \pm 1$

$\theta_p(t) = (A_1 t + A_0)\cos t + (B_1 t + B_0)\sin t$

$\theta_p'(t) = A_1\cos t - (A_1 t + A_0)\sin t + B\sin t + (B_1 t + B_0)\cot s$

$\quad = (B_1 t + A_1 + B_0)\cos t + (-A_1 t - A_0 + B_1)\sin t$

$\theta_p''(t) = B_1\cos t - (B_1 t + B_0 + A_1)\sin t - A_1\sin t + (-A_1 t - A_0 + B_1)\cos t$

$\quad = (-A_1 t - A_0 + B_1)\cos t + (-B_1 t - B_0 - 2A_1)\sin t$

Substituting into original eqn: $\theta''(t) - \theta(t)$

$\theta_p'' - \theta_p = (-A_1 t - A_0 + B_1)\cos t + (-B_1 t - B_0 - 2A_1)\sin t + (A_1 t + A_0)\cos t + (B_1 t + B_0)\sin$

$\Rightarrow -2A_1 t\cos t + (-2A_0 + 2B_1)\cos t - 2B_1 t\sin t + (-2A_1 - 2B_0)\sin t = t\sin t$

Equating Coefficients:

$-2A_1 = 0 \quad \Rightarrow \quad A_1 = 0$

$-2A_0 + 2B_1 = 0 \quad \Rightarrow \quad B_1 = A_0$

$-2B_1 = 1 \quad \Rightarrow \quad B_1 = -\frac{1}{2}$ and so, $A_0 = -\frac{1}{2}$

$-2A_1 - 2B_0 = 0 \quad \Rightarrow \quad B_0 = 0$

Therefore, the particular solution to the nonhomogeneous equation $\theta'' - \theta = t\sin t$ is given by:

$\theta_p(t) = -\frac{t\sin t + \cos t}{2}$

21.) $x''(t) - 4x'(t) + 4x(t) = te^{2t}$

$x_p = t^2(At + B)e^{2t}$

$x_p' = (3At^2 + 2Bt)e^{2t} + 2(At^3 + Bt^2)e^{2t}$

$x_p'' = (6At + 2B)e^{2t} + 4(3At^2 + 2Bt)e^{2t} + 4(At^3 + Bt^2)e^{2t}$

$(6At + 2B)e^{2t} + 4(3At^2 + 2Bt)e^{2t} + 4(At^3 + Bt^2)e^{2t} - 4[(3At^2 + 2Bt)e^{2t} + 2(At^3 + Bt^2)e^{2t}] + 4t$

$(6At + 2B)e^{2t} = te^{2t}$

$A = 0, B = 1/6$

$x_p = t^3 e^{2t}/6$

22) $x''(t) - 2x'(t) + x(t) = 24t^2 e^t$
$r^2 - 2r + 1 = 0$
$(r-1)(r-1) = 0$
$r = 1$ (double root $\Rightarrow$ $s = 2$)
$x_p = t^2(At^2 + Bt + C)e^t$
$x_p = (At^4 + Bt^3 + Ct^2)e^t$
$x_p' = (4At^3 + 3Bt^2 + 2Ct)e^t + (At^4 + Bt^3 + Ct^2)e^t$
$x_p'' = (12At^2 + 6Bt + 2C)e^t + (8At^3 + 6Bt^2 + 4Ct)e^t + (At^4 + Bt^3 + Ct^2)e^t$
Substituting into original eqn: $x''(t) - 2x'(t) + x(t)$
$(12At^2 + 6Bt + 2C)e^t + (8At^3 + 6Bt^2 + 4Ct)e^t + (At^4 + Bt^3 + Ct^2)e^t - (8At^3 + 6Bt^2 + 4Ct)e^t -$
$(12At^2 + 6Bt + 2C)e^t = 24t^2 e^t$
$12At^2 + 6Bt + 2C = 24t^2$
$12A = 24 \quad \Rightarrow \quad A = 2$
$6B = 0 \quad \Rightarrow \quad B = 0$
$2C = 0 \quad \Rightarrow \quad C = 0$
$x_p = 2t^4 e^t$