MA 221 Homework Solutions
Due date: February 24, 2009

7.2 pg. 359 # 1, 7, 9, 10, 15, 16, 17
(Underlined problems are to be handed in)

In problems 1, 7 and 9, use Definition 1 to determine the Laplace transform of the given function.

1.) t

$$L\{t\}(s) = \int_0^\infty e^{-st}dt$$

$$\lim_{N\to\infty} \int td\left(-\frac{e^{-st}}{s}\right) = \lim_{N\to\infty}\left[\left(-\frac{te^{-st}}{s}\right)_0^N + \frac{1}{s}\int_0^N e^{-st}dt\right]$$

$$\lim_{N\to\infty}\left[\left(-\frac{te^{-st}}{s}\right)_0^N + \left(\frac{e^{-st}}{s^2}\right)_0^N\right] = \lim_{N\to\infty}\left[-\frac{Ne^{-sN}}{s} + 0 - \frac{e^{-sN}}{s^2} + \frac{1}{s^2}\right]$$

$$= \frac{1}{s^2}, s > 0, e^{-sN} \to 0 \text{ and } Ne^{-sN} = \frac{N}{e^{sN}} \to 0 \text{ as } N \to \infty$$

7.) $e^{2t}\cos 3t$

$$L\{e^{2t}\cos 3t\}(s) = \int_0^\infty e^{-st}e^{2t}\cos 3tdt = \int_0^\infty e^{(2-s)t}\cos 3tdt$$

$$\lim_{N\to\infty}\left[\frac{e^{(2-s)t}((2-s)\cos 3t + 3\sin 3t)}{(2-s)^2+9}\right]_0^N$$

$$\lim_{N\to\infty}\frac{e^{(2-s)N}[(2-s)\cos 3N + 3\sin 3N] - (2-s)}{(2-s)^2+9}$$

$$\frac{s-2}{(s-2)^2+9}, s > 2$$

9.) $f(t) = \begin{cases} 0, & 0<t<2 \\ t, & 2<t \end{cases}$

$$L\{f(t)\}(s) = \int_0^\infty e^{-st}f(t)dt = \int_0^2 e^{-st}\cdot 0dt + \int_2^\infty te^{-st}dt = \int_2^\infty te^{-st}dt$$

$$\int_2^\infty te^{-st}dt = \lim_{N\to\infty}\left[\left(-\frac{te^{-st}}{s} - \frac{e^{-st}}{s^2}\right)\right]_2^N = \lim_{N\to\infty}\left[-\frac{Ne^{-sN}}{s} - \frac{e^{-sN}}{s^2} + \frac{2e^{-2s}}{s} + \frac{e^{-2s}}{s^2}\right]_0^N$$

$$\frac{2e^{-2s}}{s} + \frac{e^{-2s}}{s^2} = e^{-2s}\left(\frac{2}{s} + \frac{1}{s^2}\right)$$

$$= e^{-2s}\left(\frac{2s+1}{s^2}\right), s > 0$$

10.) $f(t) = \begin{cases} 1-t, & 0<t<1 \\ 0, & 1<t \end{cases}$

$$L\{f(t)\}(s) = \int_0^\infty e^{-st}f(t)dt = \int_0^1 e^{-st} \cdot (1-t)dt + \int_1^\infty e^{-st} \cdot 0dt = \int_0^1 (e^{-st} - te^{-st})dt$$

$$\int_0^1 (e^{-st} - te^{-st})dt = \left[\frac{1}{s^2} e^{-st}(st - s + 1) \right]_0^1$$

$$\frac{1}{s^2} e^{-s}(s - s + 1) - \left[\frac{1}{s^2}(-s+1) \right] = \frac{e^{-s}}{s^2} + \frac{1}{s} - \frac{1}{s^2}$$

$$= \frac{1}{s} + \frac{e^{-s} - 1}{s^2}, \text{for all } s$$

In problems 15, 16 and 17, use the Laplace transform table and the linearity of the Laplace transform to determine the following transforms.

15.) $L\{t^3 - te^t + e^{4t}\cos t\}$

$$L\{t^3 - te^t + e^{4t}\cos t\}(s) = L\{t^3\}(s) - L\{te^t\}(s) + L\{e^{4t}\cos t\}(s)$$

$$= \frac{3!}{s^{3+1}} - \frac{1!}{(s-1)^{1+1}} + \frac{s-4}{(s-4)^2 + 1^2}$$

$$= \frac{6}{s^4} - \frac{1}{(s-1)^2} + \frac{s-4}{(s-4)^2+1}, s > 4$$

16.) $L\{t^2 - 3t - 2e^{-t}\sin 3t\}$

$$L\{t^2 - 3t - 2e^{-t}\sin 3t\}(s) = L\{t^2\}(s) - 3L\{t\}(s) + 2L\{e^{-t}\sin 3t\}(s)$$

$$= \frac{2!}{s^{2+1}} - 3\left(\frac{1!}{s^{1+1}}\right) - 2\left(\frac{3}{(s-(-1))^2 + 3^2}\right)$$

$$= \frac{2}{s^3} - \frac{3}{s^2} - \frac{6}{(s+1)^2 + 9}, s > 0$$

17.) $L\{e^{3t}\sin 6t - t^3 + e^t\}$

$$L\{e^{3t}\sin 6t - t^3 + e^t\}(s) = L\{e^{3t}\sin 6t\}(s) - L\{t^3\}(s) + L\{e^t\}(s)$$

$$= \frac{6}{(s-3)^2 + 6^2} - \frac{3!}{s^{3+1}} + \frac{1}{s-1}$$

$$= \frac{6}{(s-3)^2 + 36} - \frac{6}{s^4} + \frac{1}{s-1}, s > 3$$