MA 221 Homework Solutions
Due Tuesday February 3, 2009

4.3, pg. 177 # 1, <u>3</u>, 5, <u>7</u>, <u>17</u>, <u>27</u>, <u>29b</u>
(Underlined problems are to be handed in)

In problems 1, 3, 5 and 7, the auxilliary equation for the given differential equation has complex roots. Find a general solution.

1.) $y'' + 9y = 0$
Auxiliary equation
$r^2 + 9 = 0$
$r = \pm 3i$
$\alpha = 0, \beta = 3$
$y(t) = c_1 e^{(0)t} \cos 3t + c_2 e^{(0)t} \sin 3t$
$y(t) = c_1 \cos 3t + c_2 \sin 3t$

<u>3</u>.) $z'' - 6z' + 10z = 0$
Auxiliary equation
$r^2 - 6r + 10 = 0$
$r = 3 \pm i$
$\alpha = 3, \beta = 1$
$z = c_1 e^{3t} \cos t + c_2 e^{3t} \sin t$

5.) $w'' + 4w' + 6w = 0$
Auxiliary equation
$r^2 + 4r + 6 = 0$
Using the quadratic equation $r = \frac{-b \pm \sqrt{b^2+4ac}}{2a}$
$r = \frac{-4 \pm \sqrt{16-24}}{2} = -2 \pm \sqrt{2}\, i$
$\alpha = -2, \beta = \sqrt{2}$
$w(t) = c_1 e^{-2t} \cos \sqrt{2}\, t + c_2 e^{-2t} \sin \sqrt{2}\, t$

<u>7</u>.) $4y'' - 4y' + 26y = 0$
Auxiliary equation
$2r^2 - 2r + 13 = 0$
$r = \frac{1}{2} \pm \frac{5}{2} i$
$\alpha = \frac{1}{2}, \beta = \frac{5}{2}$
$y(t) = c_1 e^{t/2} \cos 5t/2 + c_2 e^{t/2} \sin 5t/2$

In problem 17, find a general solution.

<u>17</u>.) $y'' - y' + 7y = 0$
Auxiliary equation
$r^2 - r + 7 = 0$
$r = \frac{1}{2} \pm \frac{3\sqrt{3}}{2} i$

$\alpha = 1/2, \beta = 3\sqrt{3}/2$
$y(t) = c_1 e^{t/2}\cos(\frac{3\sqrt{3}}{2}t) + c_2 e^{t/2}\sin(\frac{3\sqrt{3}}{2}t)$

In problem 27, solve the given initial value problem.
27.) $y''' - 4y'' + 7y' - 6y = 0, \quad y(0) = 1, \quad y'(0) = 0, \quad y''(0) = 0$
Auxiliary equation.
$r^3 - 4r^2 + 7r - 6 = 0$
Examining the divisors of –6, that is, $\pm1, \pm2, \pm3, \pm6,$ we find that $r = 2$ satisfies the equation. Next, we divide $r^3 - 4r^2 + 7r - 6 = 0$ by $r - 2$
$r^3 - 4r^2 + 7r - 6 = (r-2)(r^2 - 2r + 3) \Rightarrow r = 2; r\frac{2\pm\sqrt{4-12}}{2} = 1 \pm \sqrt{2}\,i$
A general solution to the differential equation is given by
$y(t) = c_1 e^{2t} + c_2 e^t \cos\sqrt{2}\,t + c_3 e^t \sin\sqrt{2}\,t$
$\Rightarrow y' = 2c_1 e^{2t} + c_2 e^t \cos\sqrt{2}\,t - \sqrt{2}\,c_2 e^t \sin\sqrt{2}\,t + c_3 e^t \sin\sqrt{2}\,t + \sqrt{2}\,c_3 e^t \cos\sqrt{2}\,t =$
$2e^{2t}c_1 + e^t c_2(\cos t\sqrt{2} - \sqrt{2}\sin t\sqrt{2}) + e^t c_3(\sin t\sqrt{2} + \sqrt{2}\cos t\sqrt{2});$
$y'' = 4c_1 e^{2t} + c_2 e^t(-\cos\sqrt{2}\,t - 2\sqrt{2}\sin\sqrt{2}\,t) + c_3 e^t(-\sin\sqrt{2}\,t + 2\sqrt{2}\cos\sqrt{2}\,t) \qquad \Rightarrow c_1 + c$
$2c_1 + c_2 + \sqrt{2}\,c_3 = 0;$
$4c_1 - c_2 + 2\sqrt{2}\,c_3 = 0 \qquad \Rightarrow c_1 = 1; \quad c_2 = 0; \quad c_3 = -\sqrt{2}$
$\Rightarrow y(t) = e^{2t} - \sqrt{2}\,e^t \sin\sqrt{2}\,t$

In problem 29b, find a general solution to the higher-order equation.
29.) b.
$y''' + 2y'' + 5y' - 26y = 0$
Auxiliary equation:
$r^3 + 2r^2 + 5r - 26 = 0$
Examining the divisors of –26, that is, $\pm1, \pm2, \pm13$, we find that $r = 2$ satisfies the equation. Next, we divide $y^3 + 2y^2 + 5y - 26$ by $r - 2$
$r^3 + 2r^2 + 5r - 26 = (r-2)(r^2 + 4r + 13) \qquad \Rightarrow r = 2; \quad r = -2 \pm 3i \qquad \Rightarrow$
A general solution to the differential equation is given by
$y(t) = c_1 e^{2t} + c_2 e^{-2t}\cos 3t + c_3 e^{-2t}\sin 3t$