MA 221 Homework Solutions
Due date: March 17, 2009

Section 8.2 pg. 438 # 1, 2, 5, 6

(Underlined problems are to be handed in)

1.) $\sum_{n=0}^{\infty} \frac{2^{-n}}{n+1}(x-1)^n$

$$\lim_{n\to\infty}\left(\frac{a_{n+1}}{a_n}\right) = \lim_{n\to\infty}\frac{2^{-(n+1)}/(n+2)}{2^{-n}/(n+1)} = \frac{1}{2} = L$$

$$p = \frac{1}{L} = 2$$

The endpoints of the interval of convergence are

$$x_1 = x_0 + p = 1 + 2 = 3$$

$$x_2 = x_0 - p = 1 - 2 = -1$$

At x_1

$$\sum_{n=0}^{\infty}\frac{2^{-n}}{n+1}(3-1)^n = \sum_{n=0}^{\infty}\frac{1}{n+1} = \infty$$

At x_2

$$\sum_{n=0}^{\infty}\frac{2^{-n}}{n+1}(-1-1)^n = \sum_{n=0}^{\infty}\frac{(-1)^n}{n+1} < \infty$$

Therefore, the set of convergence is

$$[-1, 3)$$

2.) $\sum_{n=0}^{\infty} \frac{3^n}{n!}x^n$

$$\lim_{n\to\infty}\left(\frac{3^{n+1}x^{n+1}}{(n+1)!} * \frac{n!}{3^n x^n}\right) = \lim_{n\to\infty}\frac{3x}{n+1} < 1$$

The set of convergence is

$$(-\infty, \infty)$$

5.) $\sum_{n=1}^{\infty} \frac{3}{n^3}(x-2)^n$

$$\lim_{n\to\infty}\left(\frac{3/(n+1)^3}{3/n^3}\right) = \lim_{n\to\infty}\left(\frac{n}{n+1}\right)^3 = 1$$

$$p = \frac{1}{L} = 1$$

At x = 3 and x = 1

$$\sum_{n=0}^{\infty} \frac{3}{n^3} \, and \sum_{n=0}^{\infty} \frac{3(-1)^n}{n^3}$$

The set of convergence is the closed interval

$$[1, 3]$$

6.) $\sum_{n=0}^{\infty} \frac{(n+2)!}{n!} (x+2)^n$

$$\lim_{n \to \infty} \left(\frac{(n+3)!}{(n+1)!} * \frac{n!}{(n+2)!} \right) = 1$$

$$\sum_{n=0}^{\infty} \frac{(n+2)!}{n!} (-1)^n \rightarrow diverges$$

$$\sum_{n=0}^{\infty} \frac{(n+2)!}{n!} (1)^n \rightarrow diverges$$

Therefore, the set of convergence is

$$(-1, -3)$$