MA 221 Homework Solutions
Due date: March 3, 2009

7.4 pg. 374 # 1, 3, 5, 7, 9, 12, 15, 17, 21, 23

(Underlined problems are to be handed in)

For problems 1,3, 5, 7, and 9, determine the inverse Laplace transform of the given function.

1.) $\frac{6}{(s-1)^4}$

From table 7.1

$$\frac{6}{(s-1)^4} = \frac{3!}{(s-1)^4}$$

is the Laplace transform of

$$e^{at}t^n$$

with

$$a = 1$$

$$n = 3$$

Therefore

$$L^{-1} = \{\frac{6}{(s-1)^4}\}(t) = e^t t^3$$

3.) $\frac{s+1}{s^2+2s+10}$

$$L^{-1}\left\{\frac{s+1}{s^2+2s+10}\right\}(t) = L^{-1}\left\{\frac{s+1}{(s+1)^2+3^2}\right\}(t)$$

From table 7.1

$$L^{-1}\left\{\frac{s+1}{(s+1)^2+3^2}\right\}$$

is the Laplace transform of

$$e^{at}\cos bt$$

with

$$a = -1$$

$$b = 3$$

Therefore

$$L^{-1} = \left\{\frac{s+1}{(s+1)^2+3^2}\right\}(t) = e^{-t}\cos 3t$$

5.) $\frac{1}{s^2+4s+8}$

$$L^{-1}\{\frac{1}{s^2+4s+8}\}(t) = L^{-1}\{\frac{1}{(s+2)^2+2^2}\}(t) = \frac{1}{2}L^{-1}\{\frac{2}{(s+2)^2+2^2}\}(t)$$
$$= \frac{1}{2}e^{-2t}\sin 2t$$

7.) $\frac{2s+16}{s^2+4s+13}$

$$\frac{2s+16}{s^2+4s+13} = \frac{2s+16}{(s+2)^2+3^2} = \frac{2(s+2)}{(s+2)^2+3^2} + \frac{4\times 3}{(s+2)^2+3^2}$$
$$2L^{-1}\{\frac{s+2}{(s+2)^2+3^2}\}(t) + 4L^{-1}\{\frac{3}{(s+2)^2+3^2}\}(t)$$
$$= 2e^{-2t}\cos 3t + 4e^{-2t}\sin 3t$$

9.) $\frac{3s-15}{2s^2-4s+10}$

$$\frac{3s-15}{2s^2-4s+10} = \frac{3}{2}\cdot\frac{s-5}{s^2-2s+5} = \frac{3}{2}\cdot\frac{(s-1)-4}{(s-1)^2+2^2} = \frac{(\frac{3}{2})(s-1)}{(s-1)^2+2^2} - \frac{3\times 2}{(s-1)^2+2^2}$$

Therefore

$$\frac{3}{2}L^{-1}\{\frac{(s-1)}{(s-1)^2+2^2}\} - 3L^{-1}\{\frac{2}{(s-1)^2+2^2}\}$$
$$\frac{3}{2}e^{t}\cos 2t - 3e^{t}\sin 2t$$

For problems 12, 15 and 17, determine the partial fraction expansions for the given rational function.

12.) $\frac{-s-7}{(s+1)(s-2)}$

$$\frac{-s-7}{(s+1)(s-2)} = \frac{A}{s+1} + \frac{B}{s-2} = \frac{A(s-2)+B(s+1)}{(s+1)(s-2)}$$

This implies

$$-s-7 = A(s-2)+B(s+1)$$

Taking $s = -1,$ and $s = 2,$ we find A, B, and C, respectively.

$$s = -1 : 1-7 = A(-1-2)+B(-1+1) \quad \Rightarrow \quad -6 = -3A$$
$$A = 2$$

$$s = 2 : -2 - 7 = A(2-2) + B(2+1) \Rightarrow \quad -9 = 3B$$

$$B = -3$$

$$\frac{2(s-2) - 3(s+1)}{(s+1)(s-2)}$$

15.) $\frac{8s - 2s^2 - 14}{(s+1)(s^2-2s+5)}$

$$\frac{8s - 2s^2 - 14}{(s+1)(s^2-2s+5)} = \frac{A}{s+1} + \frac{B(s-1) + C(2)}{(s-1)^2 + 2^2} = \frac{A[(s-1)^2 + 4] + [B(s-1) + 2C](s+1)}{(s+1)[(s-1)^2 + 4]}$$

This implies

$$-8s - 2s^2 - 14 = A[(s-1)^2 + 4] + [B(s-1) + 2C](s+1)$$

Taking $s = -1, s = 1,$ and $s = 0$ we find A, B, and C, respectively.

$$s = -1 : 8(-1) - 2(-1)^2 - 14 = A\left[(-1-1)^2 + 4\right]$$

$$A = -3$$

$$s = 1 : \quad 8(1) - 2(1)^2 - 14 = A\left[(1-1)^2 + 4\right] + 2C(1+1)$$

$$C = 1$$

$$S = 0 : \quad 8(0) - 2(0)^2 - 14 = A\left[(0-1)^2 + 4\right] + [B(0 - 1 + 2C)](0+1)$$

$$B = 1$$

$$\frac{-3}{s+1} + \frac{(s-1) + 2}{(s-1)^2 + 2^2}$$

17.) $\frac{3s+5}{s(s^2+s-6)}$

$$\frac{3s+5}{s(s^2+s-6)} = \frac{3s+5}{s(s-2)(s+3)} = \frac{A}{s} + \frac{B}{s-2} + \frac{C}{s+3}$$

$$3s + 5 = A(s-2)(s+3) + Bs(s+3) + Cs(s-2)$$

$$s = 0 : 5 = A(-2)3$$

$$A = -\frac{5}{6}$$

$$s = 2 : \quad 11 = B(2)5$$

$$C = \frac{11}{10}$$

$$S = -3 : \quad -4 = C(-3)(-5)$$

$$B = -\frac{4}{15}$$

$$= -\frac{5}{6s} + \frac{11}{10(s-2)} - \frac{4}{15(s+3)}$$

In problems 21 and 23 find $L^{-1}\{F\}$

21.) $F(s) = \frac{6s^2 - 13s + 2}{s(s-1)(s-6)}$

Since the denominator contains only nonrepeated linear factors, the partial fractions decomposition has the following form

$$\frac{6s^2 - 13s + 2}{s(s-1)(s-6)} = \frac{A}{s} + \frac{B}{s-1} + \frac{C}{s-6} = \frac{A(s-1)(s-6) + Bs(s-6) + Cs(s-1)}{s(s-1)(s-6)}$$

Evaluating both sides of the equation for $s = 0, s = 6, s = 1$:

$$s = 0 : 2 = A(-1)(-6) \Rightarrow 2 = 6A$$

$$A = \frac{1}{3}$$

$$s = 6 : \ 6s^2 - 13s + 2 = 30C = 140$$

$$C = \frac{14}{3}$$

$$s = 1 : \quad 6 - 13 + 2 = -5B = -5$$

$$B = 1$$

$$\frac{1}{3}\frac{1}{s} + \frac{1}{s-1} + \frac{14}{3}\frac{1}{s-6}$$

$$L^{-1}\left\{\frac{1}{3}\frac{1}{s} + \frac{1}{s-1} + \frac{14}{3}\frac{1}{s-6}\right\} = \frac{1}{3}L^{-1}\left\{\frac{1}{s}\right\} + L^{-1}\left\{\frac{1}{s-1}\right\} + \frac{14}{3}L^{-1}\left\{\frac{1}{s-6}\right\} = \frac{1}{3} +$$

23.) $F(s) = \frac{5s^2 - 34s + 53}{(s+3)^2(s+1)}$

In this problem, the denominator of $F(s)$ has a simple linear factor, $s + 1$, and a double linear factor, $s + 3$. Thus the decomposition has the following form

$$\frac{5s^2 - 34s + 53}{(s+3)^2(s+1)} = \frac{A}{(s+3)^2} + \frac{B}{s+3} + \frac{C}{s+1} = \frac{A(s+1) + B(s+3)(s+1) + C(s+3)^2}{(s+3)^2(s+1)}$$

Evaluating both sides of the equation for $s = -1, s = -3$:

$$s = -1 : 5 - 34 + 53 = 4C = 24$$

$$C = 6$$

$$s = -3 : \ 45 - 102 + 53 = -2A = -4$$

$$A = 2$$

$$s = 0 \text{ (To find B) :} \quad 53 = A + 3B + 9C$$

$$53 = 2 + 3B + 9(6) \Rightarrow 53 - 54 - 2 = 3B$$

$$3B = 53 - 54 - 2 \Rightarrow B = -1$$

$$2\frac{1}{(s+3)^2} - \frac{1}{s+3} + 6\frac{1}{s+1}$$

$$L^{-1}\left\{2\frac{1}{(s+3)^2} - \frac{1}{s+3} + 6\frac{1}{s+1}\right\} = 2L^{-1}\left\{\frac{1}{(s+3)^2}\right\} - L^{-1}\left\{\frac{1}{s+3}\right\} + 6L^{-1}\left\{\frac{1}{s+1}\right\} =$$