

MA 221 Homework Solutions

Due date: April 14, 2009

Section 10.2, #27 & 29

(Underlined Problems are to be handed in)

27.)

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} = 0$$

with $u(r, \theta) = R(r)T(\theta)$. Substituting into the PDE we get

$$R''T + \frac{1}{r}R'T = -\frac{1}{r^2}RT''$$

or

$$\frac{r^2R'' + rR'}{R} = -\frac{T''}{T} = \lambda$$

since the left hand side is purely a function of r , and the right hand side is purely a function of θ .

Thus we have the two ODEs

$$\begin{aligned}r^2R'' + rR' - \lambda R &= 0 \\T'' + \lambda T &= 0\end{aligned}$$

29.)

$$\frac{\partial u}{\partial t} = \beta(u_{xx} + u_{yy})$$

with $u(x, y, t) = X(x)Y(y)T(t)$. Substituting into the PDE we get

$$XYT' = \beta(X''YT + XY''T)$$

or

$$\frac{1}{\beta} \frac{T'}{T} = \frac{X''Y + XY''}{XY}$$

Since the left hand side depends only on t , whereas the right hand side depends only on x and y , then

$$\frac{1}{\beta} \frac{T'}{T} = \frac{X''Y + XY''}{XY} = K$$

where K is a constant. Therefore the equation for $T(t)$ is

$$T' - \beta KT = 0$$

Now we also have from the above equation that

$$X''Y + XY'' = KXY$$

or

$$\frac{X''}{X} = -\frac{Y''}{Y} + K$$

Since the left hand side depends only on x , whereas the right hand side depends only on y , we have

$$\frac{X''}{X} = -\frac{Y''}{Y} + K = J$$

where J is a constant. Therefore, we get the equations

$$X'' - KX = 0$$

$$Y'' + (J - K)Y = 0$$