MA 221 Homework Solutions
Due date: April 16/17, 2009

Section 10.4: 6, 8
Section10.5: 1, 3, 5
(Underlined problems are to be handed in)
6.) $f(x) = \cos x$
$0 < x < \pi$
Here $T = \pi$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{T} \text{ where } b_n = \frac{2}{T}\int_0^T f(x) \sin \frac{n\pi x}{T} dx$$

$$b_n = \frac{2}{\pi}\int_0^\pi \cos u \sin nudx$$

Substituting

$$f(x) = \sum_{k=1}^{\infty} \frac{8k}{\pi(4k^2-1)} \sin 2kx$$

8.) $f(x) = \pi - x^2$
$0 < x < \pi$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{T} \text{ where } b_n = \frac{2}{T}\int_0^T f(x) \sin \frac{n\pi x}{T} dx$$

$$b_n = \frac{2}{\pi}\int_0^\pi (\pi - x^2) \sin n\pi xdx = \frac{2}{\pi}\int_0^\pi \pi \sin n\pi xdx - \frac{2}{\pi}\int_0^\pi x^2 \sin n\pi xdx$$

$$\frac{\pi}{2} b_n = \pi \int_0^\pi \sin n\pi xdx - \int_0^\pi x^2 \sin nxdx = \frac{1}{n} - \frac{1}{n}\cos \pi^2 n - \left[[-x^2 \frac{\cos nx}{n}]_0^\pi + \frac{2}{n}\int_0^\pi x \cos nxdx \right]$$

$$= -\frac{\pi^2 \cos nx}{n} + 0 + \frac{2}{n}[(x\frac{\sin nx}{n})_0^\pi - \frac{1}{n}\int_0^\pi \sin nxdx]$$

$$= -\frac{\pi^2 \cos nx}{n} + \frac{2}{n^3}(\cos n\pi - \cos 0) = \frac{1}{n} - \frac{1}{n}\cos \pi^2 n - \frac{2}{n^3}(\cos \pi n - 1) + \frac{\pi^2}{n}\cos nx$$

$$b_n = \frac{1}{n} - \frac{\cos \pi^2 n}{n} + \frac{2\pi(-1)^{n+1}}{n} + \frac{4[(-1)^n - 1]}{\pi n^3}$$

:
Substituting

$$f(x) = \sum_{n=1}^{\infty} \frac{2}{n} \sin nx$$

Section 10.5: 1, 3, 5

In problems 1-10, find a formal solution to the given IVP

1.) $\dfrac{\partial u}{\partial t} = 5\dfrac{\partial^2 u}{\partial x^2}, \quad 0 < x < 1, \quad t > 0$

$u(0,t) = u(1,t) \quad t > 0$

$u(x,0) = (1-x)x^2, \quad 0 < x < 1$

$\beta = 5$

$L = 1$

$f(x) = (1-x)x^2$

$$u(x,t) = \sum_{n=1}^{\infty} \frac{8(-1)^{n+1} - 4}{\pi^3 n^3} e^{-5\pi^2 n^2 t^2} \sin n\pi x$$

3.) $\dfrac{\partial u}{\partial t} = 3\dfrac{\partial^2 u}{\partial x^2}, \quad 0 < x < \pi, \quad t > 0$

$u_x(0,t) = u_x(\pi,t) \quad t > 0$

$u(x,0) = x, \quad 0 < x < 1$

Let:

$\beta = 3$

$L = \pi$

$f(x) = x$

Substitute values into equation (14) on page 615:

$$u(x,t) = \sum_{n=0}^{\infty} c_n e^{-3n^2 t} \cos nx, \quad where \quad f(x) = \sum_{n=0}^{\infty} c_n \cos nx$$

Next, we find the Fourier Cosine series coefficients for $f(x) = x$

$$x = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx, \quad where$$

$a_0 = \dfrac{2}{\pi}\int_0^{\pi} x dx = \pi$

$a_n = \dfrac{2}{\pi}\int_0^{\pi} x \cos nx dx$ (integration by parts)

$a_n = \dfrac{2}{\pi}\int_0^{\pi} x \cos nx dx = a_n = \dfrac{2}{\pi}\left[\dfrac{x}{n} \sin nx|_0^{\pi} - \dfrac{1}{n}\int_0^{\pi} x \cos nx dx \right]$

$= \dfrac{2}{\pi}\left[0 - \dfrac{1}{n} \sin x (\cos nx dx)_0^{\pi} \right] = \dfrac{2}{\pi n^2}(\cos n\pi - 1) = \dfrac{2}{\pi n^2}[(-1)^n - 1]$

Thus

$$u(x,t) = \frac{\pi}{2} e^0 \cos 0 - \sum_{k=0}^{\infty} \frac{4}{\pi(2k+1)^2} e^{-3(2k+1)^2 t} \cos(2k+1)x$$

$$u(x,t) = \frac{\pi}{2} - \sum_{k=0}^{\infty} \frac{4}{\pi(2k+1)^2} e^{-3(2k+1)^2 t} \cos(2k+1)x$$

5.) $\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2}, \quad 0 < x < \pi, \quad t > 0$

$u_x(0,t) = u_x(\pi,t) = 0 \quad t > 0$

$u(x,0) = e^x, \quad 0 < x < \pi$

Let:

$\beta = 1$

$L = \pi$

$f(x) = e^x$

$$u(x,t) = \frac{2(e^{\pi} - 1)}{\pi} + \sum_{n=1}^{\infty} \frac{2e^{\pi}(-1)^n - 2}{\pi(1 + n^2)} e^{-n^2 t} \cos nx$$