Ma 227 Homework due 9/3/2009

9.3 p.521 #1, 3, 6, 7, 9, 10, 16 ; 9.2 p.512 #1, 5, 9

1.

a) $A+B=\begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}+\begin{bmatrix} -1 & 0 \\ 2 & -3 \end{bmatrix}=\begin{bmatrix} 1 & 1 \\ 5 & 2 \end{bmatrix}$

b)$3A-B=3\begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}-\begin{bmatrix} -1 & 0 \\ 2 & -3 \end{bmatrix}=\begin{bmatrix} 7 & 3 \\ 7 & 18 \end{bmatrix}$

3. Let $A=\begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}, B=\begin{bmatrix} -1 & 3 \\ 5 & 2 \end{bmatrix}.$

a) $AB=\begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -1 & 3 \\ 5 & 2 \end{bmatrix}=\begin{bmatrix} -2+20 & 6+8 \\ -1+5 & 3+2 \end{bmatrix}=\begin{bmatrix} 18 & 14 \\ 4 & 5 \end{bmatrix}$

b) $A^2=AA=\begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}=\begin{bmatrix} 4+4 & 8+4 \\ 2+1 & 4+1 \end{bmatrix}=\begin{bmatrix} 8 & 12 \\ 3 & 5 \end{bmatrix}$

c) $B^2=BB=\begin{bmatrix} -1 & 3 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} -1 & 3 \\ 5 & 2 \end{bmatrix}=\begin{bmatrix} 1+15 & -3+6 \\ -5+10 & 15+4 \end{bmatrix}=\begin{bmatrix} 16 & 3 \\ 5 & 19 \end{bmatrix}$

6.

a.) $AB=\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 3 \\ 1 & 2 \end{bmatrix}=\begin{bmatrix} 0+2 & 3+4 \\ 0+1 & 3+2 \end{bmatrix}=\begin{bmatrix} 2 & 7 \\ 1 & 5 \end{bmatrix}$

b.) $(AB)C=\begin{bmatrix} 2 & 7 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 1 & -4 \\ 1 & 1 \end{bmatrix}=\begin{bmatrix} 2+7 & -8+7 \\ 1+5 & -4+5 \end{bmatrix}=\begin{bmatrix} 9 & -1 \\ 6 & 1 \end{bmatrix}$

c.) $(A+B)C=\left(\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}+\begin{bmatrix} 0 & 3 \\ 1 & 2 \end{bmatrix}\right)\begin{bmatrix} 1 & -4 \\ 1 & 1 \end{bmatrix}=\begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & -4 \\ 1 & 1 \end{bmatrix}$

$=\begin{bmatrix} 1+5 & -4+5 \\ 2+3 & -8+3 \end{bmatrix}=\begin{bmatrix} 6 & 1 \\ 5 & -5 \end{bmatrix}$

7.

a.) $\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$ $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$ $\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2 + \ldots\ldots\ldots u_n v_n$

$\mathbf{u}^T = \begin{bmatrix} u_1 & u_2 & \cdots & u_n \end{bmatrix}$

$$\mathbf{u}^T\mathbf{v} = \begin{bmatrix} u_1 & u_2 & \cdots & u_n \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = u_1 v_1 + u_2 v_2 + \ldots\ldots\ldots u_n v_n$$

b.)

$\mathbf{v}^T = \begin{bmatrix} 2 & 3 & 5 \end{bmatrix}$ $\mathbf{v} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$ $A\mathbf{v} = \begin{bmatrix} 1 & 2 & 6 \\ -1 & 2 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 2+6+30 \\ -2+6-5 \end{bmatrix} =$

$(A\mathbf{v})^T = \begin{bmatrix} 38 & -1 \end{bmatrix}$ $\mathbf{v}^T A^T = \begin{bmatrix} 2 & 3 & 5 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 2 \\ 6 & -1 \end{bmatrix} = \begin{bmatrix} 2+6+30 & -2+6-5 \end{bmatrix}$

c.) $(A\mathbf{v})^T = \mathbf{v}^T A^T$ $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$ $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$ A, transpose:

$$\begin{bmatrix} a_{11} & a_{21} & \vdots & a_{m1} \\ a_{12} & a_{22} & \vdots & a_{m2} \\ \cdots & \ldots & \cdots & \cdots \\ a_{1n} & a_{2n} & \vdots & a_{mn} \end{bmatrix}$$

$A\mathbf{v} = \begin{bmatrix} a_{11}v_1 + a_{12}v_2 + \cdots a_{1n}v_n \\ a_{21}v_1 + a_{22}v_2 + \cdots a_{2n}v_n \\ \vdots \\ a_{m1}v_1 + a_{m2}v_2 + \cdots a_{mn}v_n \end{bmatrix}$ $A\mathbf{v}$, transpose:

$$\begin{bmatrix} v_1 a_{11} + v_2 a_{12} + \cdots a_{1n} v_n & v_1 a_{21} + v_2 a_{22} + \cdots v_n a_{2n} & \vdots & v_1 a_{m1} + \cdots v_n a_{mn} + v_2 a_{2m} \end{bmatrix}$$

$$\mathbf{v}^T A^T = : \begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix} \begin{bmatrix} a_{11} & a_{21} & \vdots & a_m \\ a_{12} & a_{22} & \vdots & a_{2m} \\ \cdots & \ldots & \cdots & \cdots \\ a_{1n} & a_{2n} & \vdots & a_{mn} \end{bmatrix} =$$

$$\begin{bmatrix} v_1 a_{11} + v_2 a_{12} + \cdots a1_n v_n & v_1 a_{21} + v_2 a_{22} + \cdots v_n a_{2n} & \vdots & v_1 a_{m1} + \cdots v_n a_{mn} + v_2 a_{m2} \end{bmatrix}$$

d.) $A = \begin{bmatrix} a_{11} & a_{12} & a_{1n} \\ a_{21} & a_{22} & a_{2n} \\ a_{m1} & a_{m2} & a_{mn} \end{bmatrix}$, transpose: $\begin{bmatrix} a_{11} & a_{21} & a_{m1} \\ a_{12} & a_{22} & a_{m2} \\ a_{1n} & a_{2n} & a_{mn} \end{bmatrix}$

$B = \begin{bmatrix} b_{11} & b_{12} & b_{1n} \\ b_{21} & b_{22} & b_{2n} \\ b_{m1} & b_{m2} & b_{mn} \end{bmatrix}$, transpose: $\begin{bmatrix} b_{11} & b_{21} & b_{m1} \\ b_{12} & b_{22} & b_{m2} \\ b_{1n} & b_{2n} & b_{mn} \end{bmatrix}$

$$AB = \begin{bmatrix} a_{11} & a_{12} & a_{1n} \\ a_{21} & a_{22} & a_{2n} \\ a_{m1} & a_{m2} & a_{mn} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{1n} \\ b_{21} & b_{22} & b_{2n} \\ b_{m1} & b_{m2} & b_{mn} \end{bmatrix} =$$

$$\begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + \cdots a_{1n}b_{m1} & a_{11}b_{12} + a_{12}b_{22} + \cdots a_{1n}b_{m2} & a_{11}b_{1n} + a_{12}b_{2n} + \cdots a_{1n}b_{mn} \\ a_{21}b_{11} + a_{22}b_{21} + \ldots b_{m1}a_{2n} & a_{21}b_{12} + a_{22}b_{22} + \cdots a_{2n}b_{m2} & a_{21}b_{1n} + a_{22}b_{2n} + \cdots b_{mn}a_{2n} \\ b_{11}a_{m1} + b_{21}a_{m2} + \cdots b_{m1}a_{mn} & b_{12}a_{m1} + b_{22}a_{m2} + \cdots a_{mn}b_{m2} & a_{1m}b_{1n} + a_{m2}b_{2n} + \cdots a_{mn}b_{mn} \end{bmatrix}$$

,

$(AB)^T$:

$$\begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + \cdots a_{1n}b_{m1} & a_{21}b_{11} + a_{22}b_{21} + \ldots b_{m1}a_{2n} & b_{11}a_{m1} + b_{21}a_{m2} + \cdots b_{m1}a_{mn} \\ a_{11}b_{12} + a_{12}b_{22} + \cdots a_{1n}b_{m2} & a_{21}b_{12} + a_{22}b_{22} + \cdots a_{2n}b_{m2} & b_{12}a_{m1} + b_{22}a_{m2} + \cdots a_{mn}b_{m2} \\ a_{11}b_{1n} + a_{12}b_{2n} + \cdots a_{1n}b_{mn} & a_{21}b_{1n} + a_{22}b_{2n} + \cdots b_{mn}a_{2n} & a_{1m}b_{1n} + a_{m2}b_{2n} + \cdots a_{mn}b_{mn} \end{bmatrix}$$

$$B^T A^T = \begin{bmatrix} b_{11} & b_{21} & b_m \\ b_{12} & b_{22} & b_{2m} \\ b_n & b_{2n} & b_{mn} \end{bmatrix} \begin{bmatrix} a_{11} & a_{21} & a_m \\ a_{12} & a_{22} & a_{2m} \\ a_n & a_{2n} & a_{mn} \end{bmatrix} =$$

$$\begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + \cdots a_{1n}b_{m1} & a_{21}b_{11} + a_{22}b_{21} + \ldots b_{m1}a_{2n} & b_{11}a_{m1} + b_{21}a_{m2} + \cdots b_{m1}a_{mn} \\ a_{11}b_{12} + a_{12}b_{22} + \cdots a_{1n}b_{m2} & a_{21}b_{12} + a_{22}b_{22} + \cdots a_{2n}b_{m2} & b_{12}a_{m1} + b_{22}a_{m2} + \cdots a_{mn}b_{m2} \\ a_{11}b_{1n} + a_{12}b_{2n} + \cdots a_{1n}b_{mn} & a_{21}b_{1n} + a_{22}b_{2n} + \cdots b_{mn}a_{2n} & a_{1m}b_{1n} + a_{m2}b_{2n} + \cdots a_{mn}b_{mn} \end{bmatrix}$$

9. $A = \begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}$ $\quad A^{-1} = \frac{1}{|A|} \begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix}$ $\quad |A| = (2)(4) - (1)(-1) = 8 + 1 = 9$

$$A^{-1} = \frac{1}{9}\begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{4}{9} & \frac{-1}{9} \\ \frac{1}{9} & \frac{2}{9} \end{bmatrix}$$

10. Let $A = \begin{bmatrix} 4 & 1 \\ 5 & 9 \end{bmatrix}$, Find A^{-1}

$$\begin{bmatrix} 4 & 1 & 1 & 0 \\ 5 & 9 & 0 & 1 \end{bmatrix}$$

$$R_2 - \frac{5}{4}R_1 : \begin{bmatrix} 4 & 1 & 1 & 0 \\ 0 & \frac{31}{4} & \frac{-5}{4} & 1 \end{bmatrix}$$

$$R_1 - \frac{4}{31}R_2 : \begin{bmatrix} 4 & 0 & \frac{36}{31} & \frac{-4}{31} \\ 0 & \frac{31}{4} & \frac{-5}{4} & 1 \end{bmatrix}$$

$$\frac{1}{4}R_1 : \begin{bmatrix} 1 & 0 & \frac{9}{31} & \frac{-1}{31} \\ 0 & \frac{31}{4} & \frac{-5}{4} & 1 \end{bmatrix}$$

$$\frac{4}{31}R_2 : \begin{bmatrix} 1 & 0 & \frac{9}{31} & \frac{-1}{31} \\ 0 & 1 & \frac{-5}{31} & \frac{4}{31} \end{bmatrix}$$

16. a) Show that $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ is singular; ie, show that A has no inverse.

Augmented matrix: $\begin{bmatrix} 2 & -1 & 1 & 1 & 0 & 0 \\ -1 & 2 & 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{bmatrix}$.

$$\frac{1}{2}R_1 : \begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ -1 & 2 & 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{bmatrix}$$

R_2 by $R_2 + R_1$; R_3 by $R_3 - R_1$:

$$\begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{3}{2} & \frac{3}{2} & \frac{1}{2} & 1 & 0 \\ 0 & \frac{3}{2} & \frac{3}{2} & -\frac{1}{2} & 0 & 1 \end{bmatrix}$$

$$\frac{2}{3}R_2 : \begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 1 & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & \frac{3}{2} & \frac{3}{2} & -\frac{1}{2} & 0 & 1 \end{bmatrix}$$

R_1 by $R_1 + \frac{1}{2}R_2; R_3$ by $R_3 - \frac{3}{2}R_2$:

$$\begin{bmatrix} 1 & 0 & 1 & \frac{2}{3} & \frac{1}{3} & 0 \\ 0 & 1 & 1 & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 \end{bmatrix}$$

But $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 3 \\ 0 & 0 & 0 \end{bmatrix} \neq I$

OR: $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$, determinant: 0 $\Rightarrow$ matrix is singular.

b) Show that $Ax = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$ has no solutions.

Set up the augmented matrix: $\begin{bmatrix} 2 & -1 & 1 & 3 \\ -1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 3 \end{bmatrix}$. Now use row operations:

$$\frac{1}{2}R_1 : \begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{2} & \frac{3}{2} \\ -1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 3 \end{bmatrix}$$

R_2 by $R_2 + R_1; R_3$ by $R_3 - R_1$: $\begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{2} & \frac{3}{2} \\ 0 & \frac{3}{2} & \frac{3}{2} & \frac{5}{2} \\ 0 & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \end{bmatrix}$

$$\frac{2}{3}R_2 : \begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{2} & \frac{3}{2} \\ 0 & 1 & 1 & \frac{5}{3} \\ 0 & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \end{bmatrix}$$

R_1 by $R_1 + \frac{1}{2}R_2; R_3$ by $R_3 - \frac{3}{2}R_2$: $\begin{bmatrix} 1 & 0 & 1 & \frac{7}{3} \\ 0 & 1 & 1 & \frac{5}{3} \\ 0 & 0 & 0 & -1 \end{bmatrix}$

Last row is saying $0x_1 + 0x_2 + 0x_3 = -1$, which is of course impossible, so there are no solutions.

c) Show that $Ax = \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix}$ has infinitely many solutions.

$\begin{bmatrix} 2 & -1 & 1 & 3 \\ -1 & 2 & 1 & 0 \\ 1 & 1 & 2 & 3 \end{bmatrix}$. $\frac{1}{2}R_1$: $\begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{2} & \frac{3}{2} \\ -1 & 2 & 1 & 0 \\ 1 & 1 & 2 & 3 \end{bmatrix}$

R_2 by $R_2 + R_1; R_3$ by $R_3 - R_1$: $\begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{2} & \frac{3}{2} \\ 0 & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \\ 0 & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \end{bmatrix}$

$\frac{2}{3}R_2$: $\begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{2} & \frac{3}{2} \\ 0 & 1 & 1 & 1 \\ 0 & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \end{bmatrix}$

R_1 by $R_1 + \frac{1}{2}R_2; R_3$ by $R_3 - \frac{3}{2}R_2$: $\begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Now if we let $x_3 = c, c$ arbitrary, then we have $x_2 = 1 - c, x_1 = 2 - c$.

Can also express solution as $x = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + c\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$.

9.2 p.512 #1, 5, 9

1. $\begin{matrix} x_1 + 2x_2 + 2x_3 = 6 \\ 2x_1 + x_2 + x_3 = 6 \\ x_1 + x_2 + 3x_3 = 6 \end{matrix} = \begin{bmatrix} 1 & 2 & 2 & 6 \\ 2 & 1 & 1 & 6 \\ 1 & 1 & 3 & 6 \end{bmatrix}$

$$R_2 - 2R_1 \quad and \quad R_3 - R_1 \begin{bmatrix} 1 & 2 & 2 & 6 \\ 0 & -3 & -3 & -6 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

$$\tfrac{-1}{3} R_2 \begin{bmatrix} 1 & 2 & 2 & 6 \\ 0 & 1 & 1 & 2 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

$$R_1 - 2R_2 \;\; and \;\; R_3 + R_2 \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 2 & 2 \end{bmatrix}$$

$$x_1 = 2 \quad x_2 + x_3 = 2 \quad 2x_3 = 2 \qquad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

5. $$\begin{aligned} -x_1 + 2x_2 &= 0 \\ 2x_1 + 3x_2 &= 0 \end{aligned} = \begin{bmatrix} -1 & 2 & 0 \\ 2 & 3 & 0 \end{bmatrix}$$

$$R_2 + 2R_1 : \begin{bmatrix} -1 & 2 & 0 \\ 0 & 7 & 0 \end{bmatrix} = \begin{aligned} -x_1 + 2x_2 &= 0 \\ 7x_2 &= 0 \end{aligned}$$

So x_2=0. Substituting 0 in for x_2 for the first equation gives x_1=0.

9.

We eliminate x_1 from the first equation by adding (1-i) times the second equation to it:

$$\begin{aligned} [2 - (1+i)(1-i)]x_2 &= 0 \\ -x_1 - (1+i)x_2 &= 0 \end{aligned}$$

Since (1-i)(1+i)=$1^2 - i^2 = 1 - (-1) = 2,$ we obtain

$$\begin{aligned} 0 &= 0 \\ -x_1 - (1+i)x_2 &= 0 \end{aligned} \quad \Rightarrow x_2 = \tfrac{-1}{1+i} x_1 = \tfrac{-1+i}{2} x_1$$

Assigning an arbitrary complex value to x_1, say 2s, we see that the system has infinitely many solutions

given by x_1=2s x_2=(-1+i)s where s is any complex number.