Ma 227 Homework 2 Solutions Fall 2009
Due 9/10/2009

9.3-pg.522 - 523
11, 13, 19, 23, 25, 28, 29

11. Find the inverse $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 3 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 2 & 3 & 2 & 0 & 0 & 1 \end{bmatrix} \Rightarrow$Inverse DoesNotExist

13. Find the inverse $A = \begin{bmatrix} -2 & -1 & 1 \\ 2 & 1 & 0 \\ 3 & 1 & -1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & -1 & 2 & 0 & 1 & 0 \\ 1 & 1 & 4 & 0 & 0 & 1 \end{bmatrix}$

R_2 by $R_2 + R_1; R_3$ by $2R_3 + 3R_1$: $\begin{bmatrix} -2 & -2 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & -1 & 1 & 3 & 0 & 2 \end{bmatrix}$

R_1 by $R_1 - R_3$: $\begin{bmatrix} -2 & 0 & 0 & -2 & 0 & -2 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & -1 & 1 & 3 & 0 & 2 \end{bmatrix}$

$\frac{1}{2}R_2; R_3 \to R_2; R_2 \to R_3$: $\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & -1 & 1 & 3 & 0 & 2 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$

R_2 by $-R_2 + R_3$: $\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -2 & 1 & -2 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$

$$\Rightarrow A^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ -2 & 1 & -2 \\ 1 & 1 & 0 \end{bmatrix}$$

19. $X(t) = \begin{bmatrix} e^{t} & e^{-t} & e^{2t} \\ e^{t} & -e^{-t} & 2e^{2t} \\ e^{t} & e^{-t} & 4e^{2t} \end{bmatrix}$. $X^{-1}(t) = ?$

$$\left[\begin{array}{ccc|ccc} e^{t} & e^{-t} & e^{2t} & 1 & 0 & 0 \\ e^{t} & -e^{-t} & 2e^{2t} & 0 & 1 & 0 \\ e^{t} & e^{-t} & 4e^{2t} & 0 & 0 & 1 \end{array}\right]. \quad R_2 \text{ by } R_2 - R_1; R_3 \text{ by } R_3 - R_1 :$$

$$\begin{bmatrix} e^{t} & e^{-t} & e^{2t} & 1 & 0 & 0 \\ 0 & -2e^{-t} & e^{2t} & -1 & 1 & 0 \\ 0 & 0 & 3e^{2t} & -1 & 0 & 1 \end{bmatrix}$$

$$e^{-t}R_1 : \begin{bmatrix} 1 & e^{-2t} & e^{t} & e^{-t} & 0 & 0 \\ 0 & -2e^{-t} & e^{2t} & -1 & 1 & 0 \\ 0 & 0 & 3e^{2t} & -1 & 0 & 1 \end{bmatrix}$$

$$-\frac{1}{2}e^{t}R_2; \frac{1}{3}e^{-2t}R_3 : \begin{bmatrix} 1 & e^{-2t} & e^{t} & e^{-t} & 0 & 0 \\ 0 & 1 & -\frac{1}{2}e^{3t} & \frac{1}{2}e^{t} & -\frac{1}{2}e^{t} & 0 \\ 0 & 0 & 1 & -\frac{1}{3}e^{-2t} & 0 & \frac{1}{3}e^{-2t} \end{bmatrix}$$

$$R_1 \text{ by } R_1 - e^{-2t}R_2 : \begin{bmatrix} 1 & 0 & \frac{3}{2}e^{t} & \frac{1}{2}e^{-t} & \frac{1}{2}e^{-t} & 0 \\ 0 & 1 & -\frac{1}{2}e^{3t} & \frac{1}{2}e^{t} & -\frac{1}{2}e^{t} & 0 \\ 0 & 0 & 1 & -\frac{1}{3}e^{-2t} & 0 & \frac{1}{3}e^{-2t} \end{bmatrix}$$

$$R_1 \text{ by } R_1 - \frac{3}{2}e^{t}R_3; R_2 \text{ by } R_2 + \frac{1}{2}e^{3t}R_3 : \begin{bmatrix} 1 & 0 & 0 & e^{-t} & \frac{1}{2}e^{-t} & -\frac{1}{2}e^{-t} \\ 0 & 1 & 0 & \frac{1}{3}e^{t} & -\frac{1}{2}e^{t} & \frac{1}{6}e^{t} \\ 0 & 0 & 1 & -\frac{1}{3}e^{-2t} & 0 & \frac{1}{3}e^{-2t} \end{bmatrix}$$

$$\Rightarrow X^{-1}(t) = \begin{bmatrix} e^{-t} & \frac{1}{2}e^{-t} & -\frac{1}{2}e^{-t} \\ \frac{1}{3}e^{t} & -\frac{1}{2}e^{t} & \frac{1}{6}e^{t} \\ -\frac{1}{3}e^{-2t} & 0 & \frac{1}{3}e^{-2t} \end{bmatrix}.$$

23. Find determinant: $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 2 \\ 1 & 5 & -2 \end{bmatrix}$

$$\begin{vmatrix} 1 & 0 & 0 \\ 3 & 1 & 2 \\ 1 & 5 & -2 \end{vmatrix} = 1\begin{vmatrix} 1 & 2 \\ 5 & -2 \end{vmatrix} - 0\begin{vmatrix} 3 & 2 \\ 1 & -2 \end{vmatrix} + 0\begin{vmatrix} 3 & 1 \\ 1 & 5 \end{vmatrix} = 1(-2-10) - 0 + 0 = -12.$$

25. Find determinant: $\begin{bmatrix} 1 & 4 & 3 \\ -1 & -1 & 2 \\ 4 & 5 & 2 \end{bmatrix}$

$$\begin{vmatrix} 1 & 4 & 3 \\ -1 & -1 & 2 \\ 4 & 5 & 2 \end{vmatrix} = 1\begin{vmatrix} -1 & 2 \\ 5 & 2 \end{vmatrix} - 4\begin{vmatrix} -1 & 2 \\ 4 & 2 \end{vmatrix} + 3\begin{vmatrix} -1 & -1 \\ 4 & 5 \end{vmatrix} = 1(-12) - 4(-10) + 3(-1) = 25$$

28. Determine the values of r for which $\det(A - rI) = 0$.

$$A = \begin{bmatrix} 3 & 3 \\ 2 & 4 \end{bmatrix} \Rightarrow \det(A - rI) = \begin{vmatrix} 3-r & 3 \\ 2 & 4-r \end{vmatrix} = (3-r)(4-r) - 6 = 0$$

$$\Rightarrow 12 - 7r + r^2 - 6 = 0$$

$$r^2 - 7r + 6 = 0$$

$$r = 6; r = 1$$

29. Determine the values of r for which $\det(A - rI) = 0$.

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}. \quad \det(A - rI) = \begin{vmatrix} -r & 0 & 0 \\ 0 & 1-r & 0 \\ 1 & 0 & 1-r \end{vmatrix} = -r(1-r)^2 = 0$$

$$\Rightarrow r_1 = r_2 = 1, r_3 = 0.$$

9.5-pg.541 3, 5, 7

3. Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$.

$$\det(A - rI) = 0 \Rightarrow \begin{vmatrix} 1-r & -1 \\ 2 & 4-r \end{vmatrix} = (1-r)(4-r) + 2 = 6 - 5r + r^2$$

$$= (r-3)(r-2) = 0 \Rightarrow r = 3, 2.$$

So the eigenvalues are 3 & 2.

Now, for each eigenvalue, solve $(A - rI)u = 0$:

$r = 3$: $\quad A - rI = \begin{bmatrix} -2 & -1 \\ 2 & 1 \end{bmatrix}$;

$(A - rI)u = 0 \Rightarrow \begin{bmatrix} -2 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Both equations imply $u_2 = -2u_1$.

Let $u_1 = s$; then $u_2 = -2s$, and the associated eigenvector is $\begin{bmatrix} s \\ -2s \end{bmatrix}$ or $s\begin{bmatrix} 1 \\ -2 \end{bmatrix}$.

$r = 2$: $\quad A - rI = \begin{bmatrix} -1 & -1 \\ 2 & 2 \end{bmatrix}$;

$(A - rI)u = 0 \Rightarrow \begin{bmatrix} -1 & -1 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Both equations imply $u_2 = -u_1$.

Let $u_1 = s$; then $u_2 = -s$, and the associated eigenvector is $\begin{bmatrix} s \\ -s \end{bmatrix}$ or $s\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

5. Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$.

$$\det(A - rI) = \begin{vmatrix} 1-r & 0 & 0 \\ 0 & -r & 2 \\ 0 & 2 & -r \end{vmatrix} = (1-r)\begin{vmatrix} -r & 2 \\ 2 & -r \end{vmatrix} - 0 + 0 = (1-r)(r^2 - 4) = (1-r)(r-2$$

Eigenvalues:$\Rightarrow r = 1, 2, -2$

Now, for each eigenvalue, solve $(A - rI)u = 0$:

$r = 1$: $\quad A - rI = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 2 \\ 0 & 2 & -1 \end{bmatrix}$

$(A - rI)u = 0 \Rightarrow$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 2 \\ 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-u_2 + 2u_3 = 0$
$2u_2 - u_3 = 0$
Solving the system of equations$\Rightarrow$ $u_2 = u_3 = 0$
We assign any value to u_1 say $u_1 = s$. Therefore:

$\Rightarrow$The associated eigenvector for $r = 1$ is $\begin{bmatrix} s \\ 0 \\ 0 \end{bmatrix}$ or $s\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$r = 2 :$ $\quad A - rI = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 2 \\ 0 & 2 & -2 \end{bmatrix}$

$(A - rI)u = 0 \Rightarrow$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 2 \\ 0 & 2 & -2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-u_1 = 0$
$-2u_2 + 2u_3 = 0$
$2u_2 - 2u_3 = 0$
Solving the system of equations$\Rightarrow$ $u_1 = 0; u_2 = u_3$
We assign any value to u_2 say $u_2 = u_3 = s$. Therefore:

$\Rightarrow$The associated eigenvector for $r = 2$ is $\begin{bmatrix} 0 \\ s \\ s \end{bmatrix}$ or $s\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

$r = -2 :$ $\quad A - rI = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 2 & 2 \end{bmatrix}$

$(A - rI)u = 0 \Rightarrow$

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 2 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$3u_1 = 0$
$2u_2 + 2u_3 = 0$
$2u_2 + 2u_3 = 0$

Solving the system of equations$\Rightarrow$ $u_1 = 0; u_2 = -u_3$
We assign any value to u_3,say $u_3 = s$. Then $u_2 = -s$

$\Rightarrow$The associated eigenvector for $r = -2$ is $\begin{bmatrix} 0 \\ -s \\ s \end{bmatrix}$ or $s\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$

7. Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 1 \\ 0 & 2 & 4 \end{bmatrix}$

$$\det(A - rI) = \begin{vmatrix} 1-r & 0 & 0 \\ 2 & 3-r & 1 \\ 0 & 2 & 4-r \end{vmatrix} = 8r^2 - 17r - r^3 + 10 = -(r-5)(r-1)(r-2)$$

$-(r-5)(r-1)(r-2) = 0$
Eigenvalues: $r = 1,2,5$
Eigenvectors:
$r = 1$:

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 2 & 2 & 1 & 0 \\ 0 & 2 & 3 & 0 \end{bmatrix} R_1 \to R_2; R_2 \to R_3 : \begin{bmatrix} 2 & 2 & 1 & 0 \\ 0 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

R_1 by $R_1 - R_2$: $\begin{bmatrix} 2 & 0 & -2 & 0 \\ 0 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow u_1 = u_3; 2u_2 = -3u_3$

Let $u_1 = s$. Then $u_2 = -\frac{3}{2}s = s \Rightarrow \vec{u} = s\begin{bmatrix} 1 \\ -\frac{3}{2} \\ 1 \end{bmatrix}$

$r = 2$:

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 \\ 0 & 2 & 2 & 0 \end{bmatrix} R_1 \to -R_1; R_2 \text{ by } R_2 - 2R_1; \tfrac{1}{2}R_3 : \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

$$R_3 \text{ by } R_3 - R_2 : \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow u_1 = 0; u_2 = -u_3$$

Let $u_3 = s$. Then $u_2 = -s \Rightarrow \vec{u} = s\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$

$r = 5$:

$$\begin{bmatrix} -4 & 0 & 0 & 0 \\ 2 & -2 & 1 & 0 \\ 0 & 2 & -1 & 0 \end{bmatrix} R_1 \to -\tfrac{1}{4}R_1; \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & -2 & 1 & 0 \\ 0 & 2 & -1 & 0 \end{bmatrix}$$

$$R_2 \text{ by } R_2 + R_3; \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 2 & -1 & 0 \end{bmatrix}; R_2 \text{ by } R_2 - 2R_1 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & -1 & 0 \end{bmatrix} \Rightarrow u_1 = 0; u_3 = 2u_2$$

Let $u_2 = s$. Then $u_3 = 2s \Rightarrow \vec{u} = s\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$