Ma 227 Homework 4 Solutions Fall 2009
Due 9/24/2009

9.5 - pg. 542 - #20,21, 32, 33

20. Find a fundamental matrix for

$$A = \begin{bmatrix} 5 & 4 \\ -1 & 0 \end{bmatrix}$$

We first calculate the eigenvalues and eigenvectors.

$$\begin{vmatrix} 5-r & 4 \\ -1 & -r \end{vmatrix} = r^2 - 5r + 4 = (r-4)(r-1)$$

Thus the eigenvalues of A are $r = 1, 4$.

The system of equations $(A - rI)X = 0$ is

$$(5-r)x_1 + 4x_2 = 0$$
$$-x_1 - rx_2 = 0$$

Putting $r = 1$ yields $x_1 = -x_2$ so the corresponding eigenvector is

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Putting $r = 4$ yields $x_1 = -4x_2$ so the corresponding eigenvector is

$$\begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

Hence the two LI solutions are

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix} e^t, \begin{bmatrix} 4 \\ -1 \end{bmatrix} e^{4t}$$

A fundamental matrix for this system is

$$\begin{bmatrix} e^t & 4e^{4t} \\ -e^t & -e^{4t} \end{bmatrix}$$

21. Find a fundamental matrix for

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 8 & -14 & 7 \end{bmatrix}$$

We first calculate the eigenvalues and eigenvectors.

$$\begin{vmatrix} -r & 1 & 0 \\ 0 & -r & 1 \\ 8 & -14 & 7-r \end{vmatrix} = -r^3 + 7r^2 - 14r + 8$$

Thus $p(r) = r^3 - 7r^2 + 14r - 8$. Note that $p(1) = p(2) = p(4) = 0$. Hence

$$p(r) = r^3 - 7r^2 + 14r - 8 = (r-1)(r-2)(r-4)$$

and the eigenvalues are $r = 1, 2, 4$.

The system $(A - rI)X = 0$ is

$$\begin{aligned} -rx_1 + x_2 + 0x_3 &= 0 \\ 0x_1 - rx_2 + x_3 &= 0 \\ 8x_1 - 14x_2 + (7-r)x_3 &= 0 \end{aligned}$$

$r = 1$ yields

$$\begin{aligned} -x_1 + x_2 &= 0 \\ -x_2 + x_3 &= 0 \\ 8x_1 - 14x_2 - 6x_3 &= 0 \end{aligned}$$

Thus $x_1 = x_2 = x_3$ and the corresponding eigenvector is

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

For $r = 2$ we get

$$\begin{aligned} -2x_1 + x_2 &= 0 \\ -2x_2 + x_3 &= 0 \\ 8x_1 - 14x_2 + 5x_3 &= 0 \end{aligned}$$

Thus $x_2 = 2x_1$ and $x_3 = 2x_2 = 4x_1$ and the corresponding eigenvector is

$$\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

For $r = 4$ the system becomes

$$-4x_1 + x_2 + 0x_3 = 0$$
$$0x_1 - 4x_2 + x_3 = 0$$
$$8x_1 - 14x_2 + 3x_3 = 0$$

Thus $4x_1 = x_2$ and $x_3 = 4x_2$. Letting $x_1 = 1$ we have the eigenvector

$$\begin{bmatrix} 1 \\ 4 \\ 16 \end{bmatrix}$$

Thus a fundamental matrix is

$$\begin{matrix} e^t & e^{2t} & e^{4t} \\ e^t & 2e^{2t} & 4e^{4t} \\ e^t & 4e^{2t} & 16e^{4t} \end{matrix}$$

.

32. Solve the given IVP

$$\vec{x}'(t) = \begin{bmatrix} 6 & -3 \\ 2 & 1 \end{bmatrix} \vec{x}(t), \qquad \vec{x}(0) = \begin{bmatrix} -10 \\ 6 \end{bmatrix}.$$

$$\det(A - rI) = \begin{vmatrix} 6 - r & -3 \\ 2 & 1 - r \end{vmatrix} = (6 - r)(1 - r) + 6 = r^2 - 7r + 12 = 0$$

$$(r - 3)(r - 4)$$

$$\Rightarrow r = 3, 4$$

$$r = 3 : \qquad u_1 = (A - 3I)u = \begin{bmatrix} 3 & -3 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$3u_1 - 3u_2 = 0$
$2u_1 - 2u_2 = 0$
Solving the system $\Rightarrow u_1 = u_2$
Setting $u_1 = s \Rightarrow u_2 = s$

$$\Rightarrow u_1 = \begin{bmatrix} s \\ s \end{bmatrix} = s \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\Rightarrow 3u_1 = 3u_2 \Rightarrow u_1 = u_2$, so eigenvector is $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$r = 4: \qquad u_2 = (A-4I)u = \begin{bmatrix} 2 & -3 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$2u_1 - 3u_2 = 0$
$2u_1 - 3u_2 = 0$
Solving the system $\Rightarrow u_1 = \frac{3}{2}u_2$
Setting $u_2 = s \Rightarrow u_1 = \frac{3}{2}s$

$$\Rightarrow u_2 = \begin{bmatrix} \frac{3}{2}s \\ s \end{bmatrix} = s\begin{bmatrix} \frac{3}{2} \\ 1 \end{bmatrix}$$

A fundamental matrix is $\begin{bmatrix} e^{3t} & \frac{3}{2}e^{4t} \\ e^{3t} & e^{4t} \end{bmatrix}$.

Then $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} e^{3t} & \frac{3}{2}e^{4t} \\ e^{3t} & e^{4t} \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$

$$\vec{x}(0) = \begin{bmatrix} 1 & \frac{3}{2} \\ 1 & 1 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -10 \\ -6 \end{bmatrix}$$

$c_1 + \frac{3}{2}c_2 = -10$
$c_1 + c_2 = -6 \Rightarrow c_1 = -6 - c_2$

$\Rightarrow -6 - c_2 + \frac{3}{2}c_2 = -10 \Rightarrow c_2 = -8; c_1 = 2$

So $\vec{x}(t) = \begin{bmatrix} e^{3t} & \frac{3}{2}e^{4t} \\ e^{3t} & e^{4t} \end{bmatrix}\begin{bmatrix} 2 \\ -8 \end{bmatrix} = \begin{bmatrix} 2e^{3t} - 12e^{4t} \\ 2e^{3t} - 8e^{4t} \end{bmatrix}$.

33. Solve the given IVP

$$\vec{x}'(t) = \begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}\vec{x}(t), \qquad \vec{x}(0) = \begin{bmatrix} -2 \\ -3 \\ 2 \end{bmatrix}.$$

$$\det(A - rI) = \begin{vmatrix} 1-r & -2 & 2 \\ -2 & 1-r & -2 \\ 2 & -2 & 1-r \end{vmatrix} = (1-r)\begin{vmatrix} 1-r & -2 \\ -2 & 1-r \end{vmatrix} + 2\begin{vmatrix} -2 & -2 \\ 2 & 1-r \end{vmatrix} + 2\begin{vmatrix} -2 & 1-r \\ 2 & -2 \end{vmatrix}$$

$$(1-r)\left[(1-r)^2-4\right]+2[-2(1-r)+4]+2[4-2(1-r)] = 0$$
$$(1-r)(r-3)(r+1)+8(r+1) = -(r+1)(r-5)(r+1)(r+1) = 0$$

$$\Rightarrow r = 5, -1(multiplicity 2)$$

$$r = 5: \qquad u_1 = (A-5I)u = \begin{bmatrix} -4 & -2 & 2 \\ -2 & -4 & -2 \\ 2 & -2 & -4 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-4u_1 - 2u_2 + 2u_3 = 0$
$-2u_1 - 4u_2 - 2u_3 = 0$
$2u_1 - 2u_2 - 4u_3 = 0$
Solving the system
$\Rightarrow 2u_1 = 2u_2 + 4u_3$
$-2u_2 - 4u_3 - 4u_2 - 2u_3 = -6u_2 - 6u_3 = 0$
$u_2 = -u_3$
$2u_1 = -2u_3 + 4u_3 \Rightarrow u_1 = u_3$
Setting $u_3 = s \Rightarrow u_2 = -s \Rightarrow u_1 = s$

$$\Rightarrow u_1 = \begin{bmatrix} s \\ -s \\ s \end{bmatrix} = s\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$r = -1: \qquad u_1 = (A+1I)u = \begin{bmatrix} 2 & -2 & 2 \\ -2 & 2 & -2 \\ 2 & -2 & 2 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2u_1 - 2u_2 + 2u_3 = 0$
$-2u_1 + 2u_2 - 2u_3 = 0$
$2u_1 - 2u_2 + 2u_3 = 0$
Simplifying the system
$u_1 - u_2 + u_3 = 0$
$u_1 + u_2 - u_3 = 0$ (last 2 eqns are the same)

Setting $u_2 = s$ and $u_3 = v$ then $u_1 = s - v$

$$\Rightarrow u_2 = \begin{bmatrix} s-v \\ s \\ v \end{bmatrix} = s\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + v\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

The General Solutions is as follows:

$$x(t) = c_1 e^{5t}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + c_2 e^{-t}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_3 e^{-t}\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}.$$

Plugging in IV

$$x(0) = c_1\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + c_2\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_3\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \\ 2 \end{bmatrix}.$$

Solving the system:

$c_1 + c_2 - c_3 = -2$
$-c_1 + c_2 = -2$
$c_1 + c_3 = 2$
$\Rightarrow c_1 = 1, c_2 = -2, c_3 = 1$

$$x(t) = e^{5t}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} - 2e^{-t}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + e^{-t}\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}.$$

$$x(t) = \begin{bmatrix} e^{5t} - 2e^{-t} - e^{-t} \\ -e^{5t} - 2e^{-t} + 0 \\ e^{5t} + 0 + e^{-t} \end{bmatrix} = \begin{bmatrix} e^{5t} - 3e^{-t} \\ -e^{5t} - 2e^{-t} \\ e^{5t} + e^{-t} \end{bmatrix}$$

9.6 - pg. 549 - 1,2,3,5, 7

Find a general solution of the system $\vec{x}' = A\vec{x}$ for the given matrix A.

1. $A = \begin{bmatrix} 2 & -4 \\ 2 & -2 \end{bmatrix}.$ Eigenvalues:

$$\det(A - rI) = \begin{vmatrix} 2-r & -4 \\ 2 & -2-r \end{vmatrix} = r^2 + 4 = 0 \Rightarrow r = \pm 2i = \alpha \pm i\beta$$

Eigenvectors:

$r = 2i$:

$$\begin{bmatrix} 2-2i & -4 \\ 2 & -2-2i \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 2-2i & -4 & 0 \\ 2 & -2-2i & 0 \end{bmatrix}$$

R_1 says $(2-2i)u_1 = 4u_2 \Rightarrow u_2 = \left(\frac{2-2i}{4}\right)u_1 = \left(\frac{1}{2} - \frac{i}{2}\right)u_1$. Let $u_1 = s$;

then $\vec{u} = \begin{bmatrix} s \\ \left(\frac{1}{2} - \frac{i}{2}\right)s \end{bmatrix} = s\begin{bmatrix} 1 \\ \frac{1}{2} \end{bmatrix} + is\begin{bmatrix} 0 \\ -\frac{1}{2} \end{bmatrix}$. Let $s = 2$:

$\Rightarrow \vec{u} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} + i\begin{bmatrix} 0 \\ -1 \end{bmatrix} = \vec{a} + i\vec{b}$.

So general solution is

$$\vec{x}(t) = c_1\left\{e^{0t}\cos 2t\begin{bmatrix} 2 \\ 1 \end{bmatrix} - e^{0t}\sin 2t\begin{bmatrix} 0 \\ -1 \end{bmatrix}\right\}$$

$$+ c_2\left\{e^{0t}\sin 2t\begin{bmatrix} 2 \\ 1 \end{bmatrix} + e^{0t}\cos 2t\begin{bmatrix} 0 \\ -1 \end{bmatrix}\right\}$$

$$= c_1\begin{bmatrix} 2\cos 2t \\ \cos 2t + \sin t2t \end{bmatrix} + c_2\begin{bmatrix} 2\sin 2t \\ \sin 2t - \cos 2t \end{bmatrix}$$

2. $A = \begin{bmatrix} -2 & -5 \\ 1 & 2 \end{bmatrix}$

$$p(r) = \det(A - rI) = \begin{vmatrix} -2-r & -5 \\ 1 & 2-r \end{vmatrix} = (r^2 - 4) + 5 = r^2 + 1$$

The eigenvalues are therefore $r = \pm i$.

The system $(A - rI)X = 0$ is

$$(-2-r)x_1 - 5x_2 = 0$$
$$x_1 + (2-r)x_2 = 0$$

Putting $r = i$ yields

$$(2+i)x_1 + 5x_2 = 0$$
$$x_1 + (2-i)x_2 = 0$$

Multiplication of the second equation by $2 + i$ yields the second equation. Thus $x_2 = -\frac{2+i}{5}x_1$.
Let $x_1 = -5$ and an eigenvector is

$$\begin{bmatrix} -5 \\ 2+i \end{bmatrix} = \begin{bmatrix} -5 \\ 2 \end{bmatrix} + i\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Here $\alpha = 0, \beta = 1, a = \begin{bmatrix} -5 \\ 2 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$\mathbf{x}_1(t) = e^{\alpha t}(\cos\beta t\mathbf{a} - \sin\beta t\mathbf{b})$$

$$\mathbf{x}_2(t) = e^{\alpha t}(\sin\beta t\mathbf{a} + \cos\beta t\mathbf{b})$$

$$\begin{aligned} x(t) &= c_1x_1(t) + c_2x_2(t) \\ &= c_1\begin{bmatrix} -5\cos t \\ 2\cos t - \sin t \end{bmatrix} + c_2\begin{bmatrix} -5\sin t \\ 2\sin t + \cos t \end{bmatrix} \end{aligned}$$

3. $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix}$

$$\det(A - rI) = \begin{vmatrix} 1-r & 2 & -1 \\ 0 & 1-r & 1 \\ 0 & -1 & 1-r \end{vmatrix} = (1-r)\begin{vmatrix} 1-r & 1 \\ -1 & 1-r \end{vmatrix} - 0 + 0 = 0$$

$$(1-r)\left[(1-r)^2 + 1\right] = 0$$

$$\Rightarrow (1-r)(r^2 - 2r + 1) = 0$$
$$\Rightarrow r = 1, 1 \pm i$$

Eigenvectors:
$r = 1$:

$$u_1 = (A - rI)u = \begin{bmatrix} 0 & 2 & -1 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2u_2 - u_3 = 0$
$u_3 = 0$
$-u_2 = 0$
Setting $u_1 = s \Rightarrow u_2 = 0 \Rightarrow u_3 = 0$

$$\Rightarrow u_1 = \begin{bmatrix} s \\ 0 \\ 0 \end{bmatrix} = s\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$r = 1+i \qquad u_2 = (A+rI)u = \begin{bmatrix} -i & 2 & -1 \\ 0 & -i & 1 \\ 0 & -1 & i \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-iu_1 + 2u_2 - u_3 = 0$
$-iu_2 + u_3 = 0$
$-u_2 + iu_3 = 0$
Setting $u_2 = s$ then $u_3 = is$

$-iu_1 + 2s - is = 0$
$(-iu_1)i = (-2s+is)i \Rightarrow (i^2 = -1)$
$u_1 = -2si - s$

$$\Rightarrow u_2 = \begin{bmatrix} -2si - s \\ s \\ is \end{bmatrix} = s\begin{bmatrix} -2i-1 \\ 1 \\ i \end{bmatrix} + i\begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

The General Solutions is as follows:

$$x(t) = c_1e^t\cos t\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + c_1e^t\sin t\begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} + c_2e^t\sin t\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + c_2e^t\cos t\begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} + c_3e^t\Big[$$

5. $A = \begin{bmatrix} -1 & -2 \\ 8 & -1 \end{bmatrix}$

$$\det\begin{bmatrix} -1-r & -2 \\ 8 & -1-r \end{bmatrix} = r^2 + 2r + 17$$

$r^2 + 2r + 17 = 0$, Solution is: $-1+4i, -1-4i.$ Thus $\alpha = -1, \beta = 4.$

The System $(A - rI)X = 0$ is

$$(-1-r)x_1 - 2x_2 = 0$$
$$8x_1 + (-1-r)x_2 = 0$$

Letting $r = -1+4i$ we get

$$(-4i)x_1 - 2x_2 = 0$$
$$8x_1 + 4ix_1 = 0$$

Multiplying the first equation by $-4i$ yields the second equation. Thus $-2ix_1 = x_2$ so an eigenvector is

$$\begin{matrix} -2i \\ 1 \end{matrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + i\begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

Hence $a = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, b = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$ and

$$x_1(t) = e^{-t}\cos 4t\begin{bmatrix} 0 \\ 1 \end{bmatrix} - e^{-t}\sin 4t\begin{bmatrix} -2 \\ 0 \end{bmatrix}$$
$$x_2(t) = e^{-t}\sin 4t\begin{bmatrix} 0 \\ 1 \end{bmatrix} + e^{-t}\cos 4t\begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

Hence a fundamental matrix is

$$\begin{bmatrix} e^{-t}\cos 4t & e^{-t}\sin 4t \\ 2e^{-t}\sin 4t & -2e^{-t}\cos 4t \end{bmatrix}$$

7. Find a fundamental matrix for the system $\vec{x}'(t) = A\vec{x}(t)$ for the given matrix A.

$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$. Eigenvalues:

$$\begin{vmatrix} -r & 0 & 1 \\ 0 & -r & -1 \\ 0 & 1 & -r \end{vmatrix} = -r^3 - r = 0 \Rightarrow r(r^2+1) = 0 \Rightarrow r = 0, \pm i.$$

Eigenvectors:

$r = 0 : \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \Rightarrow u_2 = u_3 = 0; u_1$ arbitrary. Let $u_1 = s = 1$. Then

$\vec{u} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$; corresponding solution is $e^{0t}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

$r = i$:

$$\begin{bmatrix} -i & 0 & 1 & 0 \\ 0 & -i & -1 & 0 \\ 0 & 1 & -i & 0 \end{bmatrix} \frac{R_1}{-i}, \frac{R_2}{-i} : \begin{bmatrix} 1 & 0 & i & 0 \\ 0 & 1 & -i & 0 \\ 0 & 1 & -i & 0 \end{bmatrix} \Rightarrow u_2 = iu_3;\text{ Let } u_3 = s.\text{ Then } u_2 = is;$$

$$u_1 = -iu_3 = -is \Rightarrow \vec{u} = \begin{bmatrix} -is \\ is \\ s \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} s + i \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} s.\text{ Let } s = 1.\text{ Then}$$

$$\vec{u} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + i \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = \vec{a} + i\vec{b}$$

Corresponding solutions are

$$\cos t \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - \sin t \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \sin t \\ -\sin t \\ \cos t \end{bmatrix}, \sin t \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \cos t \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -\cos t \\ \cos t \\ \sin t \end{bmatrix}$$

and a fundamental matrix is:

$$\begin{bmatrix} 1 & \sin t & -\cos t \\ 0 & -\sin t & \cos t \\ 0 & \cos t & \sin t \end{bmatrix}.$$