

Ma 227 Homework 5 Solutions Fall 2009

Due 10/15/2009

Pg. 842 Problems 5,7,21, 23

$$5) \int_0^2 \int_0^{\frac{\pi}{2}} x \sin y \, dy \, dx = \int_0^2 x \, dx \int_0^{\frac{\pi}{2}} \sin y \, dy = \left[\frac{x^2}{2} \right]_0^2 \left[-\cos y \right]_0^{\frac{\pi}{2}} = (2-0)(0+1) = 2$$

$$7) \int_0^2 \int_0^1 (2x+y)^8 \, dx \, dy = \int_0^2 \left[\frac{1}{2} \frac{(2x+y)^9}{9} \right]_{x=0}^{x=1} dy$$

Use substitution: $u = 2x + y \Rightarrow dx = \frac{1}{2} du$

$$\begin{aligned} \frac{1}{18} \int_0^2 [(2+y)^9 - (0+y)^9] dy &= \frac{1}{18} \left[\frac{(2+y)^{10}}{10} - \frac{y^{10}}{10} \right]_0^2 = \frac{1}{18} [(4^{10} - 2^{10}) - (2^{10} - 0^{10})] \\ &= \frac{1,046,528}{180} = \frac{261,632}{45} \end{aligned}$$

21) Find the volume of the solid that lies under the plane $3x + 2y + z = 12$ and above the rectangle $R = \{(x,y) | 0 \leq x \leq 1, -2 \leq y \leq 3\}$

$$\begin{aligned} z &= 12 - 3x - 2y \\ V &= \iint_R (12 - 3x - 2y) \, dA = \int_{-2}^3 \int_0^1 (12 - 3x - 2y) \, dx \, dy = \\ &= \int_{-2}^3 \left[12x - \frac{3}{2}x^2 - 2xy \right]_{x=0}^{x=1} dy = \int_{-2}^3 \left(\frac{21}{2} - 2y \right) dy \\ &= \left[\frac{21}{2}y - 2y^2 \right]_{-2}^3 = \frac{95}{2} \end{aligned}$$

23) Find the volume of the solid that lies under the elliptic paraboloid $\frac{x^2}{4} + \frac{y^2}{9} + z = 1$ and above the rectangle $R = [-1, 1] \times [-2, 2]$

$$\begin{aligned} z &= 1 - \frac{x^2}{4} - \frac{y^2}{9} \\ V &= \iint_R \left(1 - \frac{x^2}{4} - \frac{y^2}{9} \right) dV = \int_{-2}^2 \int_{-1}^1 \left(1 - \frac{x^2}{4} - \frac{y^2}{9} \right) dx \, dy = 4 \int_0^2 \int_0^1 \left(1 - \frac{x^2}{4} - \frac{y^2}{9} \right) dx \, dy \\ &= 4 \int_0^2 \left[x - \frac{1}{12}x^3 - \frac{1}{9}y^2x \right]_{x=0}^{x=1} dy = 4 \int_0^2 \left(\frac{11}{12} - \frac{1}{9}y^2 \right) dy = 4 \left[\frac{11}{12}y - \frac{1}{27}y^3 \right]_0^2 = 4 \cdot \frac{83}{54} = \frac{166}{27} \end{aligned}$$

Pg. 850 Problems 5, 7,11,15,19,33,37

$$5) \int_0^{\frac{\pi}{2}} \int_0^{\cos \theta} e^{\sin \theta} r \, dr \, d\theta = \int_0^{\frac{\pi}{2}} re^{\sin \theta} \Big|_0^{\cos \theta} d\theta = \int_0^{\frac{\pi}{2}} \cos \theta e^{\sin \theta} d\theta = e^{\sin \theta} \Big|_0^{\frac{\pi}{2}} = e - 1$$

7)

$$\iint_D x^3 y^2 dA = \int_0^2 \int_{-x}^x x^3 y^2 dy dx = \int_0^2 \left[\frac{1}{3} x^3 y^3 \right]_{-x}^x = \int_0^2 \frac{2}{3} x^6 dx = \left[\frac{2}{21} x^7 \right]_0^2 = \frac{256}{21}$$

11)

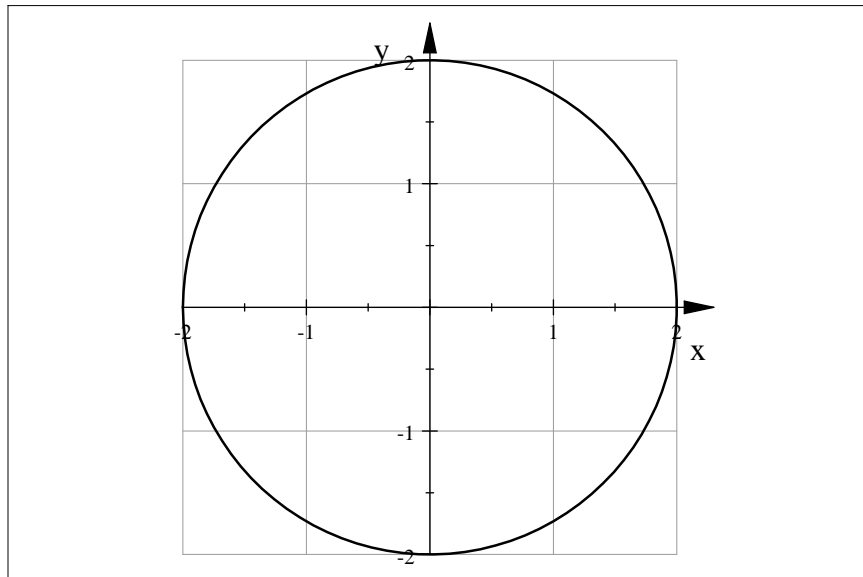
$$\begin{aligned} \iint_D x \cos y dA \quad D : y = 0, y = x^2, x = 1 \\ = \int_0^1 \int_0^{x^2} (x \cos y) dx dy \\ dx dy = \int_0^1 (x \sin y) \Big|_0^{x^2} dx = \int_0^1 (x \sin x^2) dx = -\frac{1}{2} \cos x^2 \Big|_0^1 = -\frac{1}{2} \cos(1) + \frac{1}{2} = \frac{1}{2}(1 - \cos(1)) \end{aligned}$$

15)

$$\iint_D (2x - y) dA \quad D : x^2 + y^2 = 4$$

The graph of D is shown below. Graph 2D Polare was used to get it.

$r = 2$



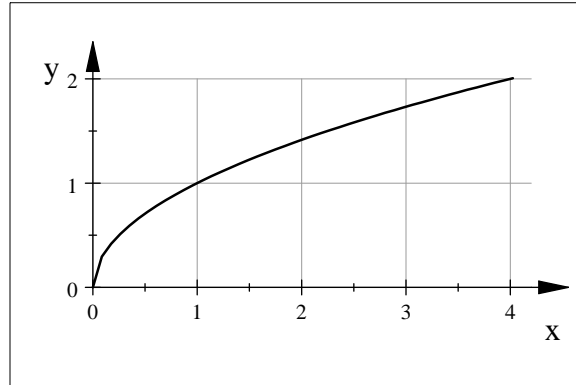
$$\begin{aligned} &= \int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (2x - y) dy dx = \int_{-2}^2 (2xy - \frac{1}{2}y^2) \Big|_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} dx \\ &= \int_{-2}^2 2x\sqrt{4-x^2} - \frac{1}{2}(4-x^2) - 2x(-\sqrt{4-x^2}) + \frac{1}{2}(4-x^2) dx \\ dx &= \int_{-2}^2 (4x\sqrt{4-x^2}) dx = \frac{4}{3}(4-x^2)^{\frac{3}{2}} \Big|_{-2}^2 = 0 \end{aligned}$$

19)

$$V = \int_1^2 \int_1^{7-3y} xy dx dy = \int_1^2 \left[\frac{1}{2} x^2 y \right]_{x=1}^{x=7-3y} dy = \frac{1}{2} \int_1^2 (48y - 42y^2 + 9y^3) dy = \frac{1}{2} [24y^2 - 14y^3 + \frac{9}{4}y^4]$$

33)

$$\sqrt{x}$$



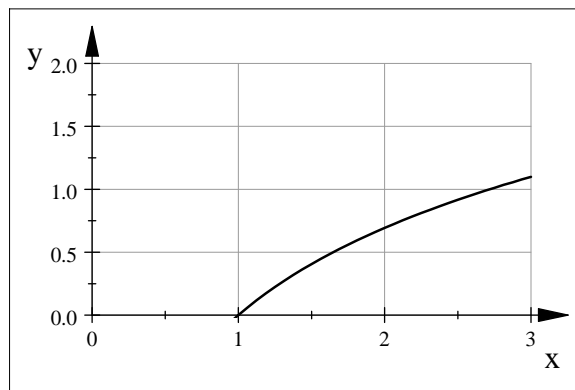
Because the region of integration is

$$D = \{(x,y) | 0 \leq y \leq \sqrt{x}, 0 \leq x \leq 4\} = \{(x,y) | y^2 \leq x \leq 4, 0 \leq y \leq 2\}$$

we have

$$\int_0^4 \int_0^{\sqrt{x}} f(x,y) dy dx = \iint_D f(x,y) dA = \int_0^2 \int_{y^2}^4 f(x,y) dx dy$$

37)



Because the region of integration is

$$D = \{(x,y) | 0 \leq y \leq \ln x, 1 \leq x \leq 2\} = \{(x,y) | e^y \leq x \leq 2, 0 \leq y \leq \ln 2\}$$

we have

$$\int_1^2 \int_0^{\ln x} f(x,y) dy dx = \iint_D y \cos x^2 dA = \int_0^{\ln 2} \int_{e^y}^2 f(x,y) dy dx$$