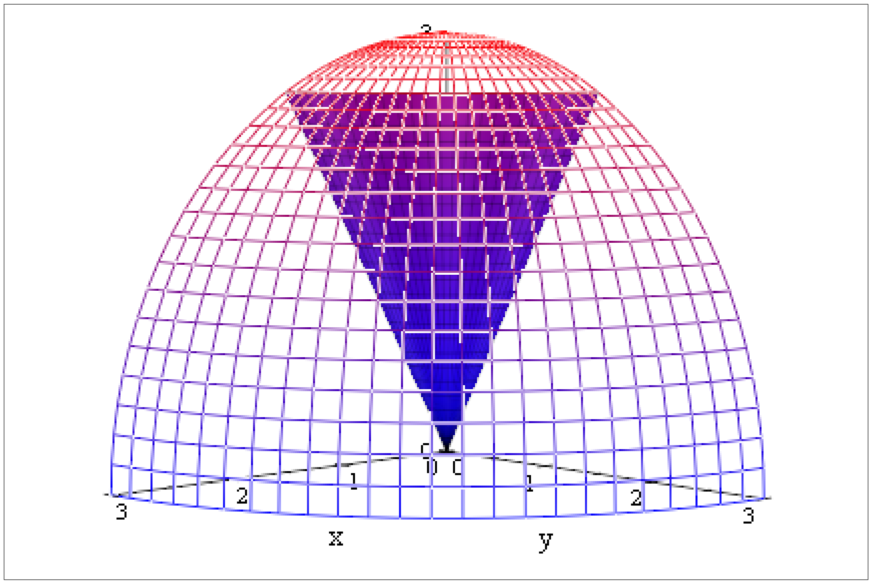
Ma 227 Homework 7 Solutions Fall 2009
Due 10/29/2009

12.8-pg. 886-887 #3, 7, 9, 11, 15, 17, 19, 21

3) $\int_0^{\frac{\pi}{6}}\int_0^{\frac{\pi}{2}}\int_0^3 \rho^2 \sin\phi\, d\rho d\theta d\phi$

The region of the integral is given by :

$E = \{(\rho,\theta,\phi)|0 \le \rho \le 3, 0 \le \theta \le \frac{\pi}{2}, 0 \le \phi \le \frac{\pi}{6}\}$. This represents the solid region in the first octant bounded above by the sphere $\rho = 3$ and below by the cone $\varphi = \frac{\pi}{6}$.



$\int_0^{\frac{\pi}{6}}\int_0^{\frac{\pi}{2}}\int_0^3 \rho^2 \sin\phi\, d\rho d\theta d\phi = (\int_0^{\frac{\pi}{6}} \sin\phi\, d\phi)(\int_0^{\frac{\pi}{2}} d\theta)(\int_0^3 d\rho) = [-\cos\phi]_0^{\frac{\pi}{6}}[\theta]_0^{\frac{\pi}{2}}[\frac{1}{3}\rho^3]_0^3 = (1 - \frac{\sqrt{3}}{2})(\frac{\pi}{2}$

7)

Evaluate $\iiint_E \sqrt{x^2+y^2}\, dV$,where E is the region that lies inside the cylinder $x^2 + y^2 = 16$ and between the planes $z = -5$ and $z = 4$.

In cylinderical coordinate E is given by

$E = \{(r,\theta,z)|0 \le \theta \le 2\pi, 0 \le r \le 4, -5 \le z \le 4\}$. So

$\iiint_E \sqrt{x^2+y^2}\, dV = \int_0^{2\pi}\int_0^4\int_{-5}^4 \sqrt{r^2}\, r dz dr d\theta = (\int_0^{2\pi} d\theta)(\int_0^4 \sqrt{r^2}\, r dr)(\int_{-5}^4 dz) = [\theta]_0^{2\pi}[\frac{1}{3}r^3]_0^4[z]_{-5}^4 =$

9) Evaluate $\iiint_E e^z dV$,where E is enclosed by the parabaloid $z = 1 + x^2 + y^2$, the cylinder $x^2 + y^2 = 5$ and the $xy-plane$

In cylinderical coordinate E is bounded by the parabaloid $z = 1 + r^2$, the cylinder $r = \sqrt{5}$ and the $xy-plane$. Thus

$\iiint_E e^z dV = \int_0^{2\pi}\int_0^{\sqrt{5}}\int_0^{1+r^2} e^z r dz dr d\theta = \int_0^{2\pi}\int_0^{\sqrt{5}} r[e^z]_{z=0}^{z=1+r^2} = \int_0^{2\pi}\int_0^{\sqrt{5}} r(e^{r^2+1} - 1) dr d\theta$

$= \int_0^{2\pi} d\theta \int_0^{\sqrt{5}} re^{r^2+1} - r dr = 2\pi\Big[\frac{1}{2}e^{1+r^2} - \frac{1}{2}r^2\Big]_0^{\sqrt{5}} = \pi(e^6 - e - 5)$

11) Evaluate $\int\int\int_E x^2 dV$,where E is the solid that lies within the cylinder $x^2+y^2 = 1,$ above the plane $z = 0$ and below the cone $z^2 = 4x^2+4y^2$

In cylidrical coordinates, E is bounded by the cylinder $r = 1$, the plane $z = 0$, and the cone $z = 2r$. So $E = \{(r,\theta,z)|0 \le \theta \le 2\pi, 0 \le r \le 1, 0 \le z \le 2r\}$ and

$\int\int\int_E x^2 dV = \int_0^{2\pi}\int_0^1\int_0^{2r} r^2\cos^2\theta r dz dr d\theta = \int_0^{2\pi}\int_0^1 [r^3\cos^2\theta z]_0^{2r} dr d\theta = \int_0^{2\pi}\int_0^1 2r^4\cos^2\theta dr d\theta = \int_0^{2\pi}$

$= \frac{2}{5}\int_0^{2\pi}\cos^2\theta d\theta = \frac{2}{5}\int_0^{2\pi}\frac{1+\cos 2\theta}{2}d\theta = \frac{1}{5}[\theta+\frac{1}{2}\sin\theta]_0^{2\pi} = \frac{2\pi}{5}.$

15) Find the mass and center of mass of the solid S bounded by the parabaloid $z = 4x^2+4y^2$ and the plane $z = a(a > 0)$ if S has a constant density K

The parabaloid $z = 4x^2+4y^2$ intersects the plane $z = a$ when $a = 4x^2+4y^2$ or $x^2+y^2 = \frac{1}{4}a.$ So in cylindrical coordinates,

$E\{(r,\theta,z)|0 \le r \le \frac{1}{2}\sqrt{a}, 0 \le \theta \le 2\pi, 4r^2 \le z \le a\}.$ Thus

$m = \int_0^{2\pi}\int_0^{\frac{\sqrt{a}}{2}}\int_{4r^2}^{a} K r dz dr d\theta = K\int_0^{2\pi}\int_0^{\frac{\sqrt{a}}{2}}(ar-4r^3)dr d\theta = K\int_0^{2\pi}[\frac{1}{2}ar^2-r^4]_{r=0}^{r=\sqrt{a}/2}d\theta = K\int_0^{2\pi}\frac{1}{16}$

Since the region is homogeneous and symmentric, $M_{yz} = M_{xz} = 0$ and

$\int_0^{2\pi}\int_0^{\frac{\sqrt{a}}{2}}\int_{4r^2}^{a} K r dz dr d\theta = K\int_0^{2\pi}\int_0^{\frac{\sqrt{a}}{2}}(\frac{1}{2}a^2r-8r^5)dr d\theta = K\int_0^{2\pi}\left[\frac{1}{4}a^2r^2-\frac{4}{3}r^6\right]_{r=0}^{r=\sqrt{a}/2}d\theta = K\int_0^{2\pi}$

$= \frac{1}{12}a^3\pi K$

Hence$(x,y,z) = (0,0,\frac{2}{3}a)$

17) In spherical coordinates, B is represented by $\{(\rho,\theta,\phi)|0 \le \rho \le 1, 0 \le \theta \le 2\pi, 0 \le \phi \le \pi\}.$ Thus

$\int\int\int_B (x^2+y^2+z^2)dV = \int_0^{\pi}\int_0^{2\pi}\int_0^1(\rho^2)\rho^2\sin\phi d\rho d\phi d\theta = \int_0^{\pi}\sin\varphi d\phi\int_0^{2\pi}d\theta\int_0^1\rho^4 d\rho = [-\cos\phi]_0^{\pi}[$

19) In spherical coordinates ,E is represented by $\{(\rho,\theta,\phi)|1 \le \rho \le 2, 0 \le \theta \le \frac{\pi}{2}, 0 \le \phi \le \frac{\pi}{2}\}$

$\int\int\int_E z dV = \int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\int_1^2(\rho\cos\phi)\rho^2\sin\phi d\rho d\phi d\theta = \int_0^{\frac{\pi}{2}}\cos\phi\sin\varphi d\phi\int_0^{\frac{\pi}{2}}d\theta\int_1^2\rho^3 d\rho = [\frac{1}{2}\sin^2\phi]_0^{\frac{\pi}{2}}[$

21.) Evaluate $\int\int\int_E x^2 dV$,where E is bounded by the $xz-plane$ and the hemispheres $y = \sqrt{9-x^2-z^2}$ and $y = \sqrt{16-x^2-z^2}$

$\int\int\int_E x^2 dV =$

$\int_0^{\pi}\int_0^{\pi}\int_3^4(\rho\sin\phi\cos\theta)^2\rho^2\sin\phi d\rho d\phi d\theta = \int_0^{\pi}\cos^2\theta d\theta\int_0^{\pi}\sin^3\phi d\phi\int_3^4\rho^4 d\rho = [\frac{1}{2}\theta+\frac{1}{4}\sin 2\theta]_0^{\pi}\left[-\frac{1}{3}\right.$

$(\frac{\pi}{2})\left(\frac{2}{3}+\frac{2}{3}\right)\frac{1}{5}(4^5-3^5) = \frac{1562}{15}\pi$

Section 12.6 Pg 870 #1, 2, 3, 5

1) Find the surface area for:
The part of the plane $z = 2 + 3x + 4y$ that lies above the rectangle $[0,5] \times [1,4]$
Here $z = f(x,y)2 + 3x + 4y$ and D is the rectangle $[0,5] \times [1,4]$, so by formula 6 the area of the surface is

$$A(S) = \iint_D \sqrt{1 + (\tfrac{\partial z}{\partial x})^2 + (\tfrac{\partial z}{\partial y})^2}\, dA = \iint_D \sqrt{1 + 3^2 + 4^2}\, dA = \sqrt{26} \iint_D dA = \sqrt{26}\, A(D) = \sqrt{26}$$

2)Find the surface area for:
The part of the plane$2x + 5y + z = 10$ that lies inside the cylinder $x^2 + y^2 = 9$
$z = f(x,y) = 10 - 2x - 5y$ and D is the disk $x^2 + y^2 \leq 9$, so by formula 6

$$A(S) = \iint_D \sqrt{1 + (\tfrac{\partial z}{\partial x})^2 + (\tfrac{\partial z}{\partial y})^2}\, dA = \iint_D \sqrt{1 + (-2)^2 + (-5)^2}\, dA = \sqrt{30} \iint_D dA = \sqrt{30}\, A(D) =$$

3)Find the surface area for:
The part of the plane $3x + 2y + z = 6$ that lies in the first octant.
$z = f(x,y) = 6 - 3x - 2y$ which intersects the xy –plane in the line $3x + 2y = 6$, so D is the triangular region given by $\{(x,y) | 0 \leq x \leq 2, 0 \leq y \leq 3 - \tfrac{3}{2}x\}$. Thus:
$A(S) = \iint_D \sqrt{1 + (-3)^2 + (-2)^2}\, dA = \sqrt{14} \iint_D dA = \sqrt{14}\, A(D) = \sqrt{14}\, (\tfrac{1}{2})(2)(3) = 3\sqrt{14}$

5) The Part of the hyperbolic parabaloid $z = y^2 - x^2$ that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$
$z = f(x,y) = y^2 - x^2$ with $1 \leq x^2 + y^2 \leq 4$. Then
$A(S) = \iint_D \sqrt{1 + 4x^2 + 4y^2}\, dA = \int_0^{2\pi} \int_1^2 r\sqrt{1 + 4r^2}\, dr d\theta = \int_0^{2\pi} d\theta \int_1^2 r\sqrt{1 + 4r^2}\, dr = [\theta]_0^{2\pi} \Big[\tfrac{1}{12} (1$