Ma 227 HW #8
Fall 2009
Due 11/05/2009

Section 13.1 - p. 910 #21, 23, 25

21.

$$f(x,y) = \ln(x+2y)$$

$$\nabla f(x,y) = f_x(x,y)\vec{i} + f_y(x,y)\vec{j}$$

$$= \frac{1}{x+2y}\vec{i} + \frac{2}{x+2y}\vec{j}$$

23. Find the gradient vector field of f

$f(x,y,z) = \sqrt{x^2+y^2+z^2}$

gradient vector field $= f_x(x,y,z)\vec{i} + f_y(x,y,z)\vec{j} + f_z(x,y,z)\vec{k}$

$$= \frac{\partial}{\partial x}\sqrt{x^2+y^2+z^2}\,\vec{i} + \frac{\partial}{\partial y}\sqrt{x^2+y^2+z^2}\,j + \frac{\partial}{\partial z}\sqrt{x^2+y^2+z^2}\,\vec{k} =$$

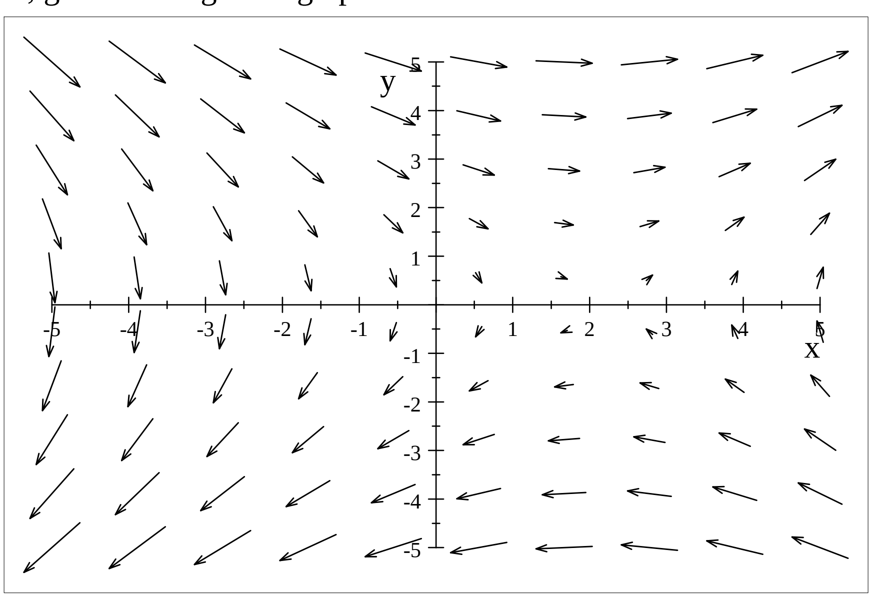
$$= \frac{x}{\sqrt{x^2+y^2+z^2}}i\_ + \frac{y}{\sqrt{x^2+y^2+z^2}}\vec{j} + \frac{z}{\sqrt{x^2+y^2+z^2}}\vec{k}$$

SNB check:

$$\nabla\left(\sqrt{x^2+y^2+z^2}\right) = \left(\frac{1}{\sqrt{x^2+y^2+z^2}}x, \frac{1}{\sqrt{x^2+y^2+z^2}}y, \frac{1}{\sqrt{x^2+y^2+z^2}}z\right)$$

25. Find the gradient vector field ∇f of f and sketch it.

$f(x,y) = xy - 2x$. Using SNB we put the cursor in the $xy - x$, then from the compute menu we select Plot 2D, gradient to get the graph below.



$\nabla(xy - 2x) = (y-2, x) = (y-2)\vec{i} + x\vec{j}.$

Section 13.5 - p. 946 #1, 5, 13, 15, 17, 19

Find (a) the curl and (b) the divergence of the vector field.

1. $F(x,y,z) = xyz\vec{i} - x^2y\vec{k}$

a. calculation of curl

$$\operatorname{curl} F = \nabla \times F = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xyz & 0 & -x^2y \end{vmatrix} = \vec{i}(-\tfrac{\partial}{\partial y}x^2y - 0) - \vec{j}(-\tfrac{\partial}{\partial x}x^2y - \tfrac{\partial}{\partial z}xyz) + \vec{k}(0 - \tfrac{\partial}{\partial y}xyz) =$$

$(-x^2) - \vec{j}(-2xy - xy) + \vec{k}(0 - xz)$

$= -x^2\vec{i} + 3xy\vec{j} - xz\vec{k}$

SNB check: $\nabla \times (e^x \sin y, e^x \cos y, z) = (0,0,0)$

b. divergence :

$\operatorname{div} F = \nabla \cdot F = \frac{\partial}{\partial x}(xyz) + \frac{\partial}{\partial y}(0) + \frac{\partial}{\partial z}(-x^2y) = yz + 0 + 0 = yz$

SNB check: $\nabla \cdot (e^x \sin y, e^x \cos y, z) = 1$

5. a. calculation of curl

$$\operatorname{curl} F = \nabla \times F = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ e^x \sin y & e^x \cos y & z \end{vmatrix} = \vec{i}(\tfrac{\partial}{\partial y}z - \tfrac{\partial}{\partial z}e^x \cos y) - \vec{j}(\tfrac{\partial}{\partial x}z - \tfrac{\partial}{\partial z}e^x \sin y) + \vec{k}(\tfrac{\partial}{\partial x}e$$

0

SNB check: $\nabla \times (e^x \sin y, e^x \cos y, z) = (0,0,0)$

b. divergence :

$\operatorname{div} F = \nabla \cdot F = \frac{\partial}{\partial x}e^x \sin y + \frac{\partial}{\partial y}e^x \cos y + \frac{\partial}{\partial z}z = 1$

SNB check: $\nabla \cdot (e^x \sin y, e^x \cos y, z) = 1$

13. Determine whether or not the vector field is conservative. If it is conservatice find the function f such that $F = \nabla f$

$\vec{F}(x,y,z) = 2xyi + (x^2 + 2yz)j + y^2k$

to check if the vector field is conservative, calculate the curl for the field.

$$curl\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy & x^2+2yz & y^2 \end{vmatrix} = (2y-2y)i-(0-0)j+(2x-2x)k = 0$$

and $\vec{F}$ is defined on all of R^3 with component functions which have continuous partial derivatives, so by Theorem 4, $\vec{F}$ is conservative. Thus there exists $f(x,y,z)$ such that $\nabla f = \vec{F}$. now

$$f_x = 2xy$$

so

$$f = x^2y + g(y,z)$$

Hence

$$f_y = x^2 + g_y(y,z) = x^2 + 2yz$$

$\Rightarrow$

$$g_y(y,z) = y^2z + h(z)$$

$\Rightarrow$

$$f = x^2y + y^2z + h(z)$$

Then

$$f_z = y^2 + h'(z) = y^2$$

$h(z) = k,$ where k is a constant. Thus

$$f(x,y,z) = x^2y + y^2z + k$$

15. Determine whether or not the vector field is conservative. If it is conservatice find the function f such that $F = \nabla f$

$\vec{F}(x,y,z) = ye^{-x}\vec{i} + e^{-x}\vec{j} + 2z\vec{k}$

calculation of curl:

$$curl\vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ ye^{-x} & e^{-x} & 2z \end{vmatrix} = \vec{i}(\tfrac{\partial}{\partial y}2z - \tfrac{\partial}{\partial z}e^{-x}) - \vec{j}(\tfrac{\partial}{\partial x}2z - \tfrac{\partial}{\partial z}ye^{-x}) + \vec{k}(\tfrac{\partial}{\partial x}e^{-x} - \tfrac{\partial}{\partial y}ye^{-x}) =$$

$-2ke^{-x} \neq 0$

which is not equal to 0, so its not conservative.

17. Is there a vector filed $\vec{G}$ on R^3 such that $curl\ \vec{G} = xy^2\vec{i} + yz^2j + zx^2k$? Explain.

Assume there is such a $\vec{G}$. Then

$\nabla \cdot \left(curl\vec{G}\right) = \frac{\partial}{\partial x}xy^2 + \frac{\partial}{\partial y}yz^2 + \frac{\partial}{\partial z}zx^2 = y^2 + z^2 + x^2$ which is not equal to 0 and hence contradicts theorem 11.

19.

Calculate the curl of $\vec{F} = f(x)\vec{i} + g(y)\vec{j} + h(z)\vec{k}.$ If this is zero, then the vector field is irrotational.

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f(x) & f(y) & f(z) \end{vmatrix} = \nabla \times (f(x), g(y), h(z)) = (0,0,0).$$ Therefore it is irrotational.