Homework #9 Ma227 Fall 2009
Due Thursday November 12, 2009

Pg 921-922
Problems: 3,5,9,17,19,25
#3
$\int_c xy^4 ds \qquad C$ is the right half of the circle $x^2 + y^2 = 16$

In this case: $\frac{-\pi}{2} \leq t \leq \frac{\pi}{2}$
$x(t) = 4\cos t \quad ; \frac{dx}{dt} = -4\sin t$
$y(t) = 4\sin t \quad ; \frac{dy}{dt} = 4\cos t$

$$\int xy^4 ds = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (4\cos t)(4\sin t)^4 \sqrt{16\sin^2 t + 16\cos^2 t}\,)dt = 4(\tfrac{4^5}{5})\sin^5 t \mid_{\frac{-\pi}{2}}^{\frac{\pi}{2}} = \frac{2(4)^6}{5} = 1638.4$$

5)
$\int_c xydx + (x-y)dy \qquad C$ consists of line segments from $(0,0)$ to $(2,0)$ and $(2,0)$ to $(3,0)$

For $0 \leq t \leq 2$: $\quad x(t) = t \quad dx = dt$
$\quad y(t) = 0 \quad dy = 0$
For $2 \leq t \leq 3$ $\quad x(t) = t \quad dx = dt$
$\quad y(t) = 2t \quad dy = 2dt$

$$\int xydx + (x-y)dy = \int_0^2 (2)(0)dt + (t)(0)dt + \int_2^3 (2t-4)tdt + (t-(2t-4)2dt = \int_2^3 (2t^2 - 6t + 8)$$

9)
$\int_c = xe^{yz}ds,\ C$ is the line segment from $(0,0,0)$ to $(1,2,3)$
$x(t) = t$
$y(t) = 2t$
$z(t) = 3t$
$0 \leq t \leq 1$

$$\int_c = xe^{yz}ds = \int_0^1 te^{(2t)(3t)}\sqrt{1^2+2^2+3^2}\,dt = \sqrt{14}\int_0^1 te^{6t^2}dt = \sqrt{14}\left[\tfrac{1}{12}e^{6t^2}\right]_0^1 = \frac{\sqrt{14}}{12}(e^6 - 1)$$

17)
$r(t) = t^2 i - t^3 j$ so $F(r(t)) = (t^2)^2(-t^3)^3 i - (-t^3)\sqrt{t^2 j} = -t^{13}i + t^4 j$
$r'(t) = 2ti - 3t^2 j$

$$\int_C \vec{F} \bullet d\vec{r} = \int_0^1 \langle F(r(t)) \bullet r'(t)dt = \int_0^1 (-2t^{14}i - 3t^6)dt = \left[-\tfrac{2}{15}t^{15} - \tfrac{3}{7}t^7\right]_0^1 = -\frac{59}{105}$$

19)

$$W = \int_C \vec{F} \bullet d\vec{r} = \int_0^1 \langle \sin t^3, \cos(-t^2), t^4\rangle \bullet \langle 3t^2, -2t, 1\rangle dt$$

$$= \int_0^1 (3t^2 \sin t^3 - 2t\cos t^2 + t^4)dt = [-\cos t^3 - \sin t^2 + \tfrac{1}{5}t^5]_0^1 = \tfrac{6}{5} - \cos 1 - \sin 1$$

25)
The part of the asteroid that lies in the quadrant is parametrized by
$x = \cos^3 t, y = \sin^3 t, 0 \le t \le \frac{\pi}{2}$
$\frac{dx}{dt} = 3\cos^2 t(-\sin t)$
$\frac{dy}{dt} = 3\sin^2 t(\cot s)$
$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{9\cos^4 t\sin^2 t + 9\sin^4 \cos^2 t} = 3\cos t\sin t\sqrt{\sin^2 t + \cos^2 t} = 3\cos t\sin t$
Therefore:
$\int_C x^3 y^5 ds = \int_0^{\frac{\pi}{2}} \cos^9 t\sin^{15} t(3\cos t\sin t)dt = \frac{945}{16{,}777{,}216}\pi$

Pg 931
Problems: 5, 9, 15,19,21
5) $\vec{F}(x,y) = \underset{P}{xe^y}\,\vec{i} + \underset{Q}{ye^x}\,\vec{j}$
$\frac{\partial P}{\partial y} = xe^y$
$\frac{\partial P}{\partial x} = ye^x$
Since $\frac{\partial Q}{\partial x} \ne \frac{\partial Q}{\partial y}$ $\vec{F}$ is **NOT** conservative

9) $\vec{F}(x,y) = \underset{P}{(ye^x + \sin y)}\,\vec{i} + \underset{Q}{(e^x + x\cos y)}\,\vec{j}$
$\frac{\partial P}{\partial y} = e^x + \cos y$
$\frac{\partial P}{\partial x} = e^x + \cos y$
F is conservative
$f_x(x,y) = ye^x + \sin y \Rightarrow f(x,y) = ye^x + x\sin y + g(y)$
$f_y(x,y) = e^x + x\cos y + g'(y)$
$f_x(x,y) = xy\cos xy + \sin xy$ so $g(x) = C$
$\Rightarrow f(x,y) = ye^x + x\sin y + C$
15)
$\vec{F}(x,y,z) = yz\,\vec{i} + xz\,\vec{j} + (xy + 2z)\vec{k}$
a)
$f_x(x,y,z) = yz \Rightarrow f(x,y,z) = yz + g(y,z)$
$f_y(x,y,z) = xz + g_y(y,z) = xz \Rightarrow g_y(y,z) = 0$
$g(y,z) = h(z)$
$f(x,y,z) = xyz + h(z); f_z(x,y,z) = xyz + h'(z) = xy + 2z \Rightarrow h'(z) = 2z \Rightarrow h(z) = z^2 + K$
$\Rightarrow$

$$f(x,y,z) = xyz + z^2 + K$$

b)
$\int_C \vec{F} \cdot d\vec{r} = f(4,6,3) - f(1,0,-2) = 81 - 4 - 77$

19) $\int \tan y dx + x\sec^2 y dy$

Let $\vec{F}(x,y) = (\tan y)\vec{i} + (\sec^2 y)\vec{j}$

Then $f(x,y) = x\tan y$ is a potential function for F so it is conservative and thus its line integral is independent of its path. Hence:

$\int_C \tan y dx + x\sec^2 y dy = \int_C F \cdot dr = f(2, \frac{\pi}{4}) - f(-1,0) = 2\tan\frac{\pi}{4} - \tan 0 = 2$

21) $\vec{F}(x,y) = (2y^{\frac{3}{2}})\vec{i} + (3x\sqrt{y})\vec{j} \qquad P(1,1), Q(2,4)$

$W = \int \vec{F} \cdot d\vec{r}$

$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} : 3\sqrt{y} = 3\sqrt{y}$

$f : \vec{F} = \nabla f$

$f(x,y) = 2xy^{\frac{3}{2}} + g(y) \Rightarrow f_y(x,y) = 3xy^{\frac{1}{2}} + g'(y).$

$f_y(x,y) = 3x\sqrt{y}$ so $g'(y) = 0 = g(y)$

$f(x,y) = 2xy^{\frac{3}{2}}$

$W = \int \vec{F} \cdot d\vec{r} = f(2,4) - f(1,1) = 2(2)(8) - 2(1) = 30$