SPECTRAL ANALYSIS OF HYPersonic BOUNDARY-LAYER INSTABILITY

by

Ahsan Hameed

A DISSERTATION

Submitted to the Faculty of the Stevens Institute of Technology
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Ahsan Hameed, Candidate

ADVISORY COMMITTEE

Nicholaus J. Parziale, Chairperson  Date

Hamid Hadim  Date

Dilhan Kalyon  Date

Jason Rabinovitch  Date

STEVENS INSTITUTE OF TECHNOLOGY
Castle Point on Hudson
Hoboken, NJ 07030
2023
SPECTRAL ANALYSIS OF HYPERSONIC BOUNDARY-LAYER INSTABILITY

ABSTRACT

The transition from laminar to turbulent flow in hypersonic boundary layers has been a topic of interest for over 50 years. Accurate prediction of the transition location is necessary to estimate the heat transferred to hypersonic vehicles and allows for optimization of the vehicle’s thermal protective systems. There are several instability phenomena that may result in the transition of a hypersonic boundary layer. In this work, the mechanisms leading to transition are experimentally investigated using focused laser differential interferometry (FLDI). Advancements to this flow diagnostic technique are made and the analytical results are validated with benchtop experiments. A method of calibration in both magnitude and frequency between the FLDI and an accelerometer is presented. Transfer functions intended to model increasingly complex disturbance fields are systematically derived. In their derivation, it is shown that strategic selection of the integration limits in the transfer functions can simplify the data-reduction process by removing the need to characterize the FLDI probe-volume length-scale.

FLDI is used in low-enthalpy and high-enthalpy facilities to investigate the effects of wall-cooling on disturbance evolution. In the low-enthalpy facility, a multi-beam pair FLDI system is used to successfully measure the disturbance phase speed. The second-mode instability is measured and shown to have higher frequency content due to wall-cooling. In the high-enthalpy facility, higher-order spectral analysis is used to investigate the nonlinear interactions between spatially separated FLDI signals located inside and outside of the boundary-layer. The cross-bicoherence analysis reveals several sum and difference interactions between the probes within the
boundary layer contributing to the generation of the first and second harmonics of the second-mode instability, providing the nonlinear mechanism for energy exchange between the second mode and its first harmonic, and modulating the second mode by low-frequency nonlinear interactions. Difference interactions between FLDI probes inside and outside of the boundary layer suggest nonlinear interactions are responsible for the exchange of energy between the second mode and the mean flow.

Author: Ahsan Hameed
Advisor: Nicholaus J. Parziale
Date: April 17, 2023
Department: Mechanical Engineering
Degree: Doctor of Philosophy
To Baba, Amma, Hadi, and Ali
Acknowledgments

First, I would like to thank my advisor, Professor Nick Parziale. His ability to foresee and mitigate the obstacles that I would encounter in my research made him an excellent manager, and his patience, support, and guidance throughout my time at Stevens made him an exceptional mentor. His strong work ethic and determination will continue to motivate and inspire me in my career.

I would like to thank my friend and lab mate, David Shekhtman. David’s diligence is only surpassed by his kindness. He was always willing to help, and provided thoughtful advice, which helped improve the quality of my work. I respect David as a scientist and a researcher and will always value his friendship. I am also grateful to have worked with the other members of the lab: Muhammad Ammar Mustafa, Roshan Adhikari, Alex Dworzanczyk, Ben Segall, and James Chen. They made the lab a fun, collaborative, and engaging work environment. I thank them for the wonderful memories they have provided me.

I wouldn’t be the person I am today without my best friend, Candace Vaughn. She provided the mature, thoughtful, and considerate voice I needed to guide me through some of the most difficult times in my life. She believed in me when I doubted myself and encouraged me to keep going. I am fortunate to have met such a wonderful person.

Finally, I would like to thank my family. My parents, Akmal Hamid and Rabia Akmal, left their families in Pakistan to provide a better life for their children. This work is, in part, an acknowledgment of the sacrifices they made for me and my brothers. Thank you for everything you have done for us, and all that you continue to do. My brothers, Ahmad and Ali Hameed, deserve my thanks and all of my love. Your smiles make it worth the effort. I am incredibly proud to be your brother.
This work is supported by the U.S. Air Force Office of Scientific Research grants FA9550-16-1-0262 and FA9550-18-1-0403.
Table of Contents

Abstract iii

Dedication v

Acknowledgments vi

List of Tables xi

List of Figures xii

1 Introduction 1

1.1 Paths to Transition 3

1.2 Mack’s Second Mode Boundary-Layer Instability 4

1.3 Sound Radiation by Supersonic Unstable Modes 9

1.4 Experimental Observations of Hypersonic Boundary-Layer Transition 12

1.5 Nonlinear Interactions in a Hypersonic Boundary Layer 14

1.6 Scope and Outline 16

2 Flow Diagnostic Technique and Flow Simulation Methods 17

2.1 Application of Laser Differential Interferometry to Hypersonic Research 17

2.2 Description of an FLDI Setup 19

2.3 Model of the FLDI and Relation to Voltage Output 21

2.4 Relation of Discrete Phase Change to Differential Phase Change via Transfer Function 25

2.5 Derivation of Transfer Functions 27

2.6 Run Condition Calculation and Stability Analysis 36
3 Benchtop Experiments Performed using FLDI

3.1 Focused Laser Differential Interferometer Response to a Controlled Phase Object

3.1.1 Experimental Setup

3.1.2 Phase Change Due to Cylindrical Lens Displacement

3.1.3 Results and Discussion

3.1.4 Conclusions

3.2 FLDI Investigation of Turbulent Jet Spectra

3.2.1 Experimental Setup

3.2.2 Results and Discussion

3.2.3 Conclusion

4 Measurement of Second-Mode Disturbances in Hypersonic Boundary Layers

4.1 Facility and Run Conditions

4.2 Experimental Setup

4.3 Stability Analysis

4.4 Results and Discussion

4.5 Conclusion

5 Nonlinear Interactions in a High-Enthalpy Boundary Layer

5.1 Facility and Experimental Setup

5.1.1 T5 Reflected-Shock Tunnel

5.1.2 FLDI Setup

5.2 Results and Discussion

5.2.1 Stability Analysis

5.2.2 Power Spectral Density
List of Tables

4.1 Shot conditions for UMD experimental campaign 60

4.2 Comparison of FLDI, schlieren, and STABL results for UMD experimental campaign 64

5.1 Reservoir conditions for Caltech experimental campaign 71

5.2 Freestream conditions for Caltech experimental campaign 72
List of Figures

1.1 Heating-rate distribution along cone for reentry F 2
1.2 General paths to turbulence in boundary layers 3
1.3 Effect of Mach number on maximum spatial amplification rates of first and second modes 5
1.4 Representation of the acoustic instability trapped within a high-speed boundary layer 6
1.5 Visualization of the second-mode instability trapped within a high-speed boundary layer 7
1.6 Spatial growth rates and phase speed for low-enthalpy flow with adiabatic wall 8
1.7 Visualization of the supersonic mode 10
1.8 Spatial growth rates and phase speed for high-enthalpy flow with cooled wall 11
2.1 General layout of an FLDI setup 20
2.2 Depiction of beam inter- and intraspacing in a multi-beam pair FLDI setup 21
2.3 Representation of FLDI beam pairs at spatial origin 22
2.4 Representation of density disturbance fields 33
2.5 Representative grid for nozzle used in nozzle simulations 37
3.1 Schematic and beam profile for single-beam pair FLDI instrument used in controlled phase object experiments 41
### 3.2 Apparatus used to measure the response of the FLDI instrument to a changing optical path length

43

### 3.3 Optical path lengths of two FLDI beams as they pass through a lens

44

### 3.4 Accelerometer response and comparison of numerical integration and FFT pre-multiplication

46

### 3.5 Phase change as measured by the accelerometer and the FLDI instrument as a function of time and frequency

46

### 3.6 Phase change as measured by an accelerometer and an FLDI instrument with a $f = -50$ mm diverging lens as a phase object

48

### 3.7 Ratio of the FLDI-derived phase change to accelerometer-derived phase-change with varying locations from the focus

49

### 3.8 Comparison of accelerometer-derived phase change to two-beam pair FLDI-derived phase change at the focus

49

### 3.9 Schematic of FLDI setup used to probe the exit of a turbulent jet

50

### 3.10 FLDI beam pairs for a setup developed using diffractive optics and a 2 arcminute Wollaston prism

51

### 3.11 Convective velocities and fits of dispersion relation for experiments with 0.5, 1, and 2 arcminute Wollaston prisms

52

### 3.12 Comparison of calculated transfer functions using a 0.5 arcminute Wollaston prism

53

### 3.13 One-dimensional energy spectra of density fluctuations of a turbulent jet

55

### 4.1 Schematic of University of Maryland’s HyperTERP shock tunnel

58

### 4.2 Typical reservoir pressure trace for HyperTERP

59

### 4.3 Temperature profile along actively cooled cone at UMD

59

### 4.4 Components of FLDI setup used at University of Maryland

61
4.5 Pictures of beams at focus for UMD experiments

4.6 Linear stability diagrams and growth rate curves for UMD shot 10 and shot 18

4.7 Second mode and broadband turbulence convective velocities using FLDI

4.8 Spectra of second mode and broadband turbulence measured by FLDI probes

4.9 Spectra of second mode and broadband turbulence measured by FLDI probes

4.10 Comparison between a cold-wall and room temperature shot at the same Reynolds number

5.1 FLDI diagnostic used at California Institute of Technology

5.2 Location of FLDI beams relative to cone surface

5.3 Reservoir pressure trace for shot 2990

5.4 Stability analysis for shot 2990

5.5 Spatial growth rate and phase speed for shot 2990

5.6 Averaged spectrograms and PSD for shot 2990

5.7 Cross-bicoherence spectrum for FLDI probes during $2335 \mu s \leq t_1 \leq 2380 \mu s$

5.8 Demodulation of FLDI signals within boundary layer

5.9 Cross-bicoherence spectrum for FLDI probes during $1805 \mu s \leq t_2 \leq 1850 \mu s$
Chapter 1

Introduction

It has been more than 50 years since William J. Knight flew a North American X-15 to Mach 6.7 above Southern California. By the time he approached Edwards Air Force Base, traveling 500 km in just 10 minutes, the extreme heat caused by the shock waves around the hypersonic vehicle had melted the support structure used to attach a mock ramjet engine to the aircraft. Though Major Knight survived the flight, the ramjet engine fell out of the California sky and served as a stark reminder of the complexity and dangers of hypersonic flight [1]. Since then, an interdisciplinary team of researchers have used theory, experiments, and computational simulations to achieve a greater understanding of the physics involved in hypersonic flight.

One topic that remains of interest is boundary-layer transition on hypersonic vehicles. Following the onset of boundary-layer transition, there is a drastic increase in aeroheating and viscous shear stress experienced by the vehicle. Fig. 1.1 shows the heating-rate distribution during the Reentry F test of a 13 ft long, 5° half-angle cone ballistic reentry vehicle (RV). The vehicle experienced a peak Mach number of approximately 20 and a total enthalpy of approximately 18 MJ/kg. After transition onset at $x/L = 0.65$, the surface heat transfer rate of the ballistic RV increased by a factor of 5. With the transition location prescribed in the computational model to match measured flight data, researchers have been able to adequately model the laminar and turbulent heating rates experienced by the ballistic RV. Heating rates were estimated with moderate accuracy in both the laminar (15-20% error) and turbulent boundary layer (20-25% error).
Figure 1.1: Heating-rate distribution along cone for Reentry-F test of a ballistic reentry vehicle [2].


Although researchers have been successful in modeling the heating rates in laminar and turbulent hypersonic boundary layers, there remains considerable uncertainty in predicting the location of boundary-layer transition [2]. In a report of the National Aerospace Plane (a national project of the 1980s to create a single-stage-to-orbit spacecraft intended for hypersonic flight) the reviewers noted a 60% error in estimating the location of transition along the body [3]. In a review of flight data of boundary-layer transition at hypersonic speeds, Schneider [4] noted a factor of 3 uncertainty in the transition location. The unpredictable onset of boundary-layer transition has resulted in conservative and ineffective hypersonic vehicle design. To ensure the integrity of the vehicle during prolonged hypersonic flight, the vehicle’s thermal protection system is specified with large factors of safety, unnecessarily adding weight, and reducing the maximum payload of the vehicle. Predicting the
onset of transition can lead to an optimization of the vehicle’s thermal protection system, improving the efficiency and effectiveness of the hypersonic vehicle.

1.1 Paths to Transition

The laminar to turbulent transition of boundary layers is a multi-step process. The main transition paths to turbulence are depicted in Fig. 1.2.

![Figure 1.2: General paths to turbulence in boundary layers](image)

Environmental disturbances that may cause the boundary layer to transition can take many forms. Natural environmental disturbances, such as the freestream noise present in a real flow field or the roughness of a surface, tend to be smaller and proceed through the traditional paths to turbulence (paths A, B, C). Path A is the typical path to turbulence in low-disturbance environments, associated with
boundary-layer instabilities such as Mack’s first and second modes, cross-flow instability, and Görtler vortices. The transient growth in path B results in higher initial eigenmode amplitudes upon crossing into an exponentially unstable region and path C is associated with transient growth that arises through the nonorthogonal nature of the eigenfunctions [6]. Larger disturbances, such as steps in the surface intended to “trip” the boundary layer, bypass the traditional transition mechanisms (paths D and E).

The receptivity of the boundary layer to disturbances is dependent on the disturbance’s form and frequency, the boundary-layer profile, and the flow conditions. Disturbances tuned for the boundary layer are received and processed, and only those considered unstable persist and are amplified within the boundary layer. The growth of the disturbance through the previously described paths leads to flow instabilities, resulting in breakdown of the flow and transition to turbulence.

1.2 Mack’s Second Mode Boundary-Layer Instability

Boundary-layer transition in hypersonic flow at zero angle of attack is affected by receptivity to freestream disturbances leading to the growth of dominant eigenmodes known as Mack’s first and second modes [7]. The first-mode instability is similar in nature to Tollmien-Schlichting (T-S) waves, whose excitation and downstream amplification Tollmien [8] and Schlichting [9][11] described as the dominant transition mechanism in low-speed (subsonic and moderate supersonic) boundary layers. In hypersonic flow, Stetson [12] suggested these T-S waves may be present in regions of relatively low Mach number, such as the tip of blunt cones. The viscous first mode is stabilized with increasing Mach number whereas the inviscid, acoustic, two-dimensional second-mode instability is amplified as its growth rate exceeds that of
the first mode with increasing Mach number \[13\]. As shown in Fig. 1.3, it becomes the dominant instability in the hypersonic regime for Mach numbers greater than 4 for insulated surfaces, and lower Mach numbers for cooled surfaces \[5\].

Figure 1.3: Effect of Mach number on maximum spatial amplification rates of first and second modes for insulated surfaces \[13\].

Due to the high edge Mach numbers in hypersonic flow, there exists a region within the boundary layer of supersonic mean flow relative to the disturbance velocity. In this region, the boundary layer acts as an acoustic waveguide. Acoustic instabilities such as the Mack second, third, and higher modes become trapped within the boundary layer, reflecting between the wall and the relative sonic line \[5, 14–16\]. As depicted in the diagram by Fedorov \[5\] (Fig. 1.4), the relative sonic line occurs at position \((y_a)\), where the local velocity \((U)\) equals the difference between the disturbance phase speed \((c)\) and the local sound speed \((a)\).
Knisely \cite{17} suggested the use of the complex local relative Mach number (\( \bar{M}(y) \)) as a useful parameter to describe the speed of propagation of the disturbance relative to the mean flow, defining it as

\[
\bar{M}(y) = \frac{\bar{u}(y) - c}{\bar{a}(y)}, \tag{1.2.1}
\]

where \( \bar{u}(y) \) is the local mean flow velocity tangential to the wall, \( c \) is the complex disturbance phase speed, and \( \bar{a}(y) \) is the local mean flow speed of sound \cite{17}. In Knisely’s representation of Fedorov’s diagram of trapped acoustic waves, shown in Fig. \ref{fig:Fig1.5}, the sonic line \( (y_s) \) occurs at \( \bar{M}(y_s) = -1 \). Below the sonic line, referred to as the supersonic region, the disturbance phase speed is supersonic with respect to the local mean flow velocity and the solution to the boundary layer stability equations is acoustic-like \cite{17}. Above the relative sonic line, the disturbance propagates subsonically with respect to the freestream and forms “rope-like” structures centered about the critical layer, denoted as \( y_c \) in Fig. \ref{fig:Fig1.5}. The critical layer occurs when the disturbance phase speed equals the local mean flow velocity, or \( \bar{M}(y_c) = 0 \).
Mack’s first and second modes are considered discrete modes and exist simultaneously in the flow with continuous modes. The discrete modes are further characterized by Fedorov and Tumin \cite{18} as the slow mode (mode $S_1$) or an infinite sequence of fast modes (mode $F_1$, mode $F_2$, etc.) based on their asymptotic behavior at the leading edge. The phase speed of the slow mode tends to $c_r = 1 - 1/M_e$ and the phase speed of the fast mode approaches $c_r = 1 + 1/M_e$, where $M_e$ is the Mach number at the boundary-layer edge. The continuous modes are the slow and fast acoustic, entropy, and vorticity disturbances, though the entropy and vorticity spectra are indistinguishable \cite{7}.

The interactions between the discrete slow and fast modes is shown in Fig. \ref{fig:stability_diagram}, which presents the stability diagram for a low-enthalpy flow with an adiabatic wall. At the leading edge, mode $S_1$ and mode $F_1$ begin with a phase speed $c_r = 1 - 1/M_e$ and $c_r = 1 + 1/M_e$, respectively. The proximity of mode $S_1$ to the slow acoustic spectrum causes it to become unstable, generating Mack’s first mode. As described above, this mode is completely stabilized at high Mach numbers. Progressing down-
stream in Fig. 1.6, mode $S_1$ increases in phase speed while mode $F_1$ decreases in phase speed. When the phase speed of mode $F_1$ equals 1, it synchronizes with external entropy/vorticity waves, facilitating the receptivity to freestream turbulence and temperature spottiness. In free-flight and quiet wind tunnels, where the freestream acoustic field can be ignored, this receptivity mechanism can be dominant [5].

Further downstream, mode $S_1$ and mode $F_1$ synchronize and achieve the same phase speed. This synchronization typically causes mode $S_1$ to become unstable while mode $F_1$ is stabilized. The traditional second mode occurs when mode $S_1$ becomes unstable after synchronization, resulting in a subsonic phase speed, $1 - 1/M_e < c_r < 1 + 1/M_e$ [7]. Mode $S_1$ can continue to synchronize with mode $F_2$, mode $F_3$, and higher modes to form Mack’s third, fourth, and higher modes.

Figure 1.6: Spatial growth rates (top) and phase speed (bottom) for low-enthalpy flow with $M_e = 4.5$, $Re = 2000$, $T_e = 65.15$ K, adiabatic wall [19].
1.3 Sound Radiation by Supersonic Unstable Modes

In cold wall, high-enthalpy flow conditions, where the wall temperature remains ambient with respect to a hot freestream, mode $F_1$ can become the unstable mode following its synchronization with mode $S_1$ and its phase speed can become supersonic ($c_r < 1 - 1/M_e$) [7]. Downstream of this synchronization, mode $F_1$ is referred to as the supersonic mode. Fig. 1.7 provides a schematic of the supersonic mode, as visualized by Knisely and Zhong [7]. The features in the supersonic region near the wall and the subsonic region are identical to those present in Knisely’s representation of the traditional, subsonic second mode (see Fig. 1.5). However, past the critical layer, there exists another relative sonic line and region of supersonic flow.

The second relative sonic line occurs when $\bar{M}(y_{s2}) = 1$. A second supersonic region forms above this sonic line, in which the phase speed of the disturbance travels supersonically upstream with respect to the freestream. The solution to the boundary layer stability equations is wave-like in this region, and the decaying acoustic waves are slanted at the Mach wave angle ($\mu \approx \arcsin(1/\bar{M})$) [7]. These oscillatory waves radiate into the freestream, a phenomenon referred to as the spontaneous radiation (emission) of sound by Chuvakhov and Fedorov [20].
Figure 1.7: Visualization of the supersonic mode featuring a second relative sonic line and region of supersonic flow [17].


Presented in Fig. 1.8 the stability diagram for a high-enthalpy flow with a cold wall shows distinct differences compared to the stability diagram for a low-enthalpy flow with an adiabatic wall. Particularly, mode $F_1$ becomes unstable following its synchronization with mode $S_1$, a behavior suggested by Fedorov and Khoklov dependent on whether the dispersion curve passes above or below branch points in the complex plane [19]. Additionally, the maximum growth rate of the second-mode instability for the cold wall case is two times greater, consistent with the fact that the second-mode instability is destabilized by wall cooling.
Figure 1.8: Spatial growth rates (top) and phase speed (bottom) for high-enthalpy flow with $M_e = 4.5$, $Re=2000$, $T_e = 1500$ K, $T_w = 300$K [19].

The supersonic mode has been studied since the 1980s. A thorough review of developments regarding the supersonic mode’s effect on transition in hypersonic boundary layers is provided by Knisely and Zhong [7, 17], and a summary of relevant advances follows. Initial investigations by Mack [15] and Reshotko [16] suggested that the supersonic mode played an insignificant role in hypersonic boundary-layer transition due to its smaller amplification rate compared to the second-mode instability. However, Bitter and Shepherd’s [19] work suggesting the presence of the supersonic mode in flow conditions typical of California Institute of Technology’s (Caltech) T5 reflected shock tunnel renewed interest. Using linear stability theory (LST) to study the stability of hypervelocity boundary layers over a flat plate with a sharp leading edge and very high levels of wall cooling ($T_W/T_E << 1$), Bitter and Shepherd provided evidence of the supersonic mode’s existence at hypersonic Mach numbers, and showed this mode caused the second-mode instability to remain unstable over a wider
band of frequencies compared to the subsonic mode. Knisely and Zhong investigated
Mach 5 flow over a highly-cooled (case 1: $T_W/T_E = 0.2$ and case 2: $T_W/T_E = 0.667$),
5° half-angle blunt cone with a 1 mm nose radius using LST [17] and direct numerical
simulation (DNS) [7]. Their work confirmed the existence of the supersonic mode
using both DNS and LST for case 1, but they were able to resolve the weak super-
sonic mode at the higher temperature ratio only through DNS. They confirmed the
destabilizing effects of the cold wall on the supersonic mode as suggested by Bitter
and Shepherd [19] and Chuvakhov and Fedorov [20].

The supersonic mode presents many opportunities in the understanding of
hypersonic boundary-layer transition. By radiating energy away from the boundary
layer, it may have stabilizing effects on the second-mode instability [20]. This concur-
rently could affect the energy transfer to the vehicle surface. While many researchers
have theoretically and computationally verified the existence of the supersonic mode
and its radiative characteristics, at present, the author is unaware of any experimental
study that has validated the efforts of the past 40 years.

1.4 Experimental Observations of Hypersonic Boundary-Layer Transi-
tion

The study of boundary-layer transition begins in the late 19th century with the
formative investigations of Lord Rayleigh [21], Osbourne Reynolds [22], and Ludwig
Prandtl [23]. Experimental efforts followed to support the theoretical and analytical
advancements made in the field. Schubauer and Skramstad conducted experiments
using hot wire anemometry to measure sinusoidal velocity fluctuations in the laminar
boundary layer of a flat plate [24]. Their observations confirmed the calculations
made by Tollmien [8] and Schlichting [9 11] using stability theory, who suggested the
amplification and growth of small velocity disturbances of particular wavelengths led to breakdown and boundary-layer transition. Following the successful experimental validation of the Tollmien-Schlichting theory, researchers continued using stability theory to predict the transition of boundary layers, and additional experimental and numerical studies were performed to support the pursuit of transonic and supersonic vehicles.

Since its theoretical conception, the second-mode instability has been experimentally observed using various flow measurement techniques. Demetriades [25] performed experiments at $M_\infty = 8$ over a sharp 5° half-angle cone, capturing the rope-like nature of the second-mode instability using shadowgraphs. Demetriades concluded that the waves were concentrated near the boundary-layer edge and were “made up of density crests and valleys as opposed to velocity fluctuations.” Kendall [26] performed constant-current hot-wire anemometry experiments at $M_\infty = 8.5$ over a sharp, 4° half-angle cone and measured signatures of large amplitude, periodic waves, which he identified with second-mode instability. In another set of experiments, Demetriades [27] estimated the frequency of the second-mode instability, $f_0$, to be proportional to the boundary-layer edge velocity, $U_E$, and inversely proportional to the boundary-layer thickness, $\delta$. More recently, Laurence [28] applied image processing techniques to high-speed schlieren sequences to obtain structural and propagation characteristics of the second-mode instability waves. Parziale et al. [29, 30] demonstrated the ability to measure the second-mode instability in a reflected-shock tunnel by using focused laser differential interferometry (FLDI) to measure the instability waves prior to transition at hypervelocity conditions on a slender body in the T5 reflected-shock tunnel.
1.5 Nonlinear Interactions in a Hypersonic Boundary Layer

Although the second-mode instability has been extensively studied by researchers, the nonlinear interactions between the second-mode instability and other disturbances are not well understood. Nonlinear interactions between boundary-layer disturbances leads to the formation of phase-coupled waves, resulting in the transition to turbulence due to a redistribution of spectral energy [31]. Using hot-wire anemometry data from experiments performed on a sharp cone at $M_\infty = 7.95$ by Stetson et al. [32] in AEDC Tunnel B, Kimmel and Kendall [33] found evidence of nonlinear wave propagation not accounted for by linear stability theory. They used bicoherence analysis to show phase coherence between the second-mode instability and its first harmonic and suggested nonlinear interactions between the second mode and low frequency disturbances modulated frequencies near the second mode. They noted that the measured amplification rates deviated from linear stability theory, supporting their hypothesis that the first harmonic was generated by a phase-coupled, nonlinear interaction.

Chokani [34] made similar observations in the developing boundary layer over a sharp cone generated in the low disturbance level freestream of a quiet tunnel. Chokani’s analysis suggested the low-frequency nonlinear interactions with the second-mode instability preceded the final breakdown to turbulence, noting that an initial difference interaction occurred within the spectral sidebands of the second mode prior to its low frequency modulation. Chokani [35] later expanded his analysis with the application of cross-bicoherence, allowing him to characterize the transition process as a series of discrete sum and difference nonlinear interactions between the second-mode instability, the mean flow, or the second-mode’s harmonics. He also quantified the destabilizing effect of wall cooling on the transitioning boundary layer by ob-
serving nonlinear interactions between the Mack mode and its harmonic to occur further upstream on a cooled wall than on an adiabatic wall. Using a demodulation technique, he confirmed the low-frequency amplitude modulation previously observed by Kimmel and Kendall [33]. However, the cross-bicoherence analysis performed by Chokani was based on data that was not acquired simultaneously at two spatially separated points, causing ambiguity in the observed difference interactions.

Shiplyuk et al. [36] identified the existence of a new nonlinear interaction, namely the resonance of the second mode’s subharmonic, to occur at the initial stage of the laminar-turbulent transition. Experiments performed by Bountin et al. [31] for a Mach number of $M_\infty = 5.95$ with artificial wave packets generated at the frequency of the second-mode instability used to destabilize the flow suggested subharmonic resonance to be the dominant mechanism of nonlinear interaction at the location of maximum root mean square fluctuation in a boundary layer. They also identified low-frequency nonlinear processes at the boundary-layer edge, and suggested that at the late stages of transition, nonlinear processes reach beyond the boundary layer, beginning the formation of a turbulent boundary layer. Although previous experimental efforts used hot-wire anemometry to make off-body measurements, Craig et al. [37] used high-frequency focused schlieren deflectometry along with bispectral analysis to show second-mode harmonic generation through self-interactions, and interactions with low-frequency waves resulting in amplitude modulation. Direct numerical simulations [38, 39] provided supporting evidence of fundamental and subharmonic resonance in the nonlinear transition process, and suggested that fundamental resonance may cause the breakdown to turbulence in hypersonic boundary layers over a cone.
1.6 Scope and Outline

Disturbances in high-speed boundary layers are characterized in this work using FLDI. Measurements of boundary-layer instabilities are made at two facilities: University of Maryland’s HyperTERP reflected-shock tunnel and California Institute of Technology’s T5 reflected-shock tunnel. A 5° half-angle cone is used as the test article in all experimental campaigns, and tests are performed with various configurations of nose bluntness and degrees of wall cooling.

In Chapter 2, the FLDI flow diagnostic technique is described in detail, including the development of a mathematical model for the FLDI instrument. An explanation of the methodology used to calculate experimental run conditions and perform stability analysis is also discussed. In Chapter 3, the results from benchtop experiments used to validate the FLDI technique are presented, and the new data-reduction techniques presented in Chapter 2 are shown to have merit when applied to resolving the flowfield of a turbulent free-jet. In Chapter 4, results from experiments performed at the HyperTERP reflected-shock tunnel are presented, with highlights being the measurement of disturbance phase speed within the boundary layer and validation of the effects of wall cooling on hypersonic boundary-layer transition. In Chapter 5, results from an experimental campaign at the T5 reflected-shock tunnel are presented, with a focus on the identification of nonlinear interactions between disturbances.
Chapter 2
Flow Diagnostic Technique and Flow Simulation Methods

This chapter presents the historical applications of laser differential interferometry (LDI) to study hypersonic flow. As focused laser differential interferometry (FLDI) is the flow diagnostic technique used in this research, a detailed description of its setup is provided, and a mathematical model of the FLDI is developed. A methodology for the development of transfer functions used in the reduction of data collected using an FLDI instrument is provided, and new transfer functions modeling isotropic turbulence are introduced. Following the discussion of the FLDI instrument, a description of the process used to simulate the flowfields of the experiments performed as part of this research is provided, including a discussion of the software used to perform hypersonic boundary layer stability analysis.

2.1 Application of Laser Differential Interferometry to Hypersonic Research

FLDI is an optical diagnostic technique developed by Smeets [40-45] and Smeets and George [46] in the 1970s at the French-German Research Institute of Saint-Louis (ISL). In their published work, they demonstrate the use of FLDI to measure freestream absolute density in shock tubes, develop a multiple beam FLDI system to measure density distributions of flow fields and boundary layers, exploit the focusing nature of the FLDI technique to measure the turbulent shear layer of a cold, subsonic jet, and use the interferometer as a microphone to measure the pressure profiles of acoustic waves.

While FLDI had already been introduced, laser differential interferometry
(LDI), was mostly used by researchers over the next few decades in supersonic, hypersonic, and hypervelocity flow research. Laderman and Demetriades [47] made use of the inherent divergence of a laser beam to develop an LDI setup, measuring the intensity fluctuations caused by optical gradients in the flow to detect the transition of a Mach 3 boundary layer. Azzazy et al. [48] developed a variation of the Smeets and George interferometer using a Pockels cell to compensate for the model’s vibration. In their setup, the interfering beams were fired directly onto the model surface and reflected to the signal collection optics using a variety of reflecting surfaces. They used the interferometer to calculate the dominant frequency of disturbances leading to transition in a supersonic wind tunnel. Similarly, Salyer et al. [49] added active phase compensation to their LDI setup to perform calibrated bow shock receptivity measurements in a Mach 4 quiet-flow Ludwieg tube.

Parziale [29, 30, 50–54] revitalized interest in the FLDI technique in the early 2010s. Parziale found the FLDI’s high sensitivity to boundary-layer disturbances, featuring high temporal (>10 MHz) and spatial (<1 mm) resolution, and its ability to make localized measurements well-suited for performing boundary-layer instability measurements in the high-enthalpy, hypersonic T5 reflected-shock tunnel at Caltech. He used the instrument to characterize the tunnel’s freestream disturbance level and was able to measure the second-mode boundary layer instability. Using a multi-beam pair FLDI setup, with one FLDI beam pair placed approximately 0.1 m upstream of the other, he was able to track the evolution of the second-mode instability along the model, finding the most amplified frequency measured by the downstream detector to be lower than that measured by the upstream detector, a result consistent with the hypothesis that as the boundary layer grows in size, the most amplified frequency should decrease correspondingly ($f \propto U_E/2\delta$). Knowing the spacing between the detectors and the time at which the signal was registered by each probe, Parziale
et al. were able to determine the group velocity of the narrowband second-mode disturbance to be nearly equal to the edge velocity of the boundary layer.

Since then, researchers have made additional advancements to the FLDI technique, including making reliable convective velocity measurements between two closely spaced FLDI probe volumes [55–62], facility disturbance-level characterization [63–65], and novel beam shaping techniques for application in hard-to-access flows [66–70]. Additionally, researchers have devised controlled problems [71–73] to test the data-reduction strategies formulated by Fulghum [74], Settles and Fulghum [75], and Schmidt and Shepherd [76].

2.2 Description of an FLDI Setup

The layout of an FLDI instrument is presented in Fig. 2.1. An FLDI instrument is developed by passing a linearly polarized laser beam through a diverging lens, which expands it to a large diameter at the boundary of the measurement volume. A converging lens collects the expanding beam and brings it to a sharp focus within the test region. A polarizer and birefringent prism are placed in the path of the expanding beam at (or close to) the focal length of the converging lens to generate a pair of collimated beams of mutually orthogonal linear polarization. The setup is symmetric about the focus; the beams are recombined by means of a second polarizer and a complementary birefringent prism of equivalent divergence angle, and the interference signal is measured by a change in intensity on a photodetector. This single-beam pair FLDI setup can be expanded using additional beam-splitting optics upbeam of the focus to generate multiple beam pairs, and corresponding photodetectors downbeam of the focus to measure the interference between each beam pair.
The FLDI is a common-path polarization interferometer, as the orthogonally polarized beams comprising each pair of interfering beams share a common optical path throughout most of their propagation distance except near the beam’s focus, where the beam paths differ slightly and are separated by the beam intraspacing, $\Delta x$. The beam intraspacing is dependent on the selected divergence angle of the upbeam birefringent prism and the positioning of the prism with respect to the converging lens. Wollaston prisms are typically chosen to generate the interfering beam pairs, however, researchers have successfully implemented Sanderson prisms in their FLDI setups [74]. Birefringent prisms may also be used to form a grid of multiple beam pairs interspaced in the streamwise ($x$) and wall-normal ($y$) directions, however, diffractive optical elements are able to more efficiently generate additional beam pairs and have recently come into favor [77]. The beam inter- and intraspacing is pictorially represented in Fig. 2.2.
2.3 Model of the FLDI and Relation to Voltage Output

In an FLDI system, two beams traverse closely spaced paths (in this case, the \( z \)-direction - see Fig. 2.3), are mixed with a polarization optic, and are then registered at a photodetector. The voltage response from the photodetector, \( V_D \), is the integrated intensity over the sensor face,

\[
V_D = I_D R_S R_L = R_S R_L \int_{A_S} I_D(x,y) dA,
\]

(2.3.1)

where \( I_D(x,y) \), \( I_D \), \( R_S \), \( R_L \), and \( A_S \) are the intensity at the detector face, integrated intensity, the responsivity of the photodetector, the load resistance, and the sensor area, respectively. At the photodetector face, the intensity of the optical signal generated by the FLDI beams can be related to the phase change as

\[
\bar{I}_D = \bar{I}_1 + \bar{I}_2 + 2\sqrt{\bar{I}_1 \bar{I}_2} \cos(\Delta \phi),
\]

(2.3.2)

where \( \bar{I}_1 \) and \( \bar{I}_2 \) are the integrated intensity of each FLDI beam. Assuming \( \bar{I}_1 = \bar{I}_2 = \bar{I}_0/2 \), where \( \bar{I}_0 \) is the initial intensity of the beam at the outlet of the laser, and
shifting the instrument by $\pi/2$ to the middle of a fringe, Eq. 2.3.2 reduces to

$$I_D = \bar{I}_D + \bar{I}_0 \sin(\Delta \phi).$$

(2.3.3)

Following Eq. 2.3.1, the voltage measured by the photodetector when the instrument is at the middle of a fringe is $V_0 = \bar{I}_0 R_S R_L$. Eq. 2.3.3 can then be rewritten to relate the voltage response to the phase change as

$$\Delta \phi = \sin^{-1} \left( \frac{V_D - V_0}{V_0} \right).$$

(2.3.4)

Having related the voltage and phase change, the aim is now to relate density disturbance to phase change. As a density disturbance field passes through the FLDI beams, these closely spaced beams traverse different optical path lengths (OPL). This difference in optical path lengths results in a phase difference between the beams of the FLDI instrument, which is expressed as

$$\Delta \phi = \frac{2\pi}{\lambda} (OPL_1 - OPL_2) = \frac{2\pi}{\lambda} \left( \int_{s_1} n(x, y, z) dz - \int_{s_2} n(x, y, z) dz \right),$$

(2.3.5)
where \( n(x, y, z) \) is the index of refraction of the flow field, \( \lambda \) is the wavelength of the laser, and \( s_1 \) and \( s_2 \) are the paths of the two FLDI beams. However, in practice, the voltage change measured by the photodetector is also dependent on the local intensity. For an FLDI instrument, this means that changes in index of refraction that occur at higher levels of intensity contribute more to the phase difference measured by the photodetector, and so the spatial distribution of intensity must be accounted for. As in Fig. 2.3, each FLDI beam is displaced from the origin along the ordinate by half the beam intraspacing, \( \Delta x \), as \( I \left( x \pm \frac{\Delta x}{2}, y, z \right) \). Mathematically, the change in phase is modified by introducing the local intensity of each beam into Eq. 2.3.5 via multiplication by \( A_1 = A_2 = 1 \) as

\[
\Delta \phi = \frac{2\pi}{\lambda} \left[ A_1 \int_{s_1} n(x, y, z) \, dz - A_2 \int_{s_2} n(x, y, z) \, dz \right] \quad (2.3.6)
\]

where,

\[
A_1 = \left( \frac{\int \int \int \int \int I(\frac{x - \Delta x}{2}, y, z) \, dx \, dy}{\int \int \int \int \int I(\frac{x - \Delta x}{2}, y, z) \, dx \, dy} \right)
\]

\[
A_2 = \left( \frac{\int \int \int \int \int I(\frac{x + \Delta x}{2}, y, z) \, dx \, dy}{\int \int \int \int \int I(\frac{x + \Delta x}{2}, y, z) \, dx \, dy} \right).
\]

To model the beam intensity, a Gaussian beam profile is assumed, given by

\[
I(x, y, z) = \frac{2}{w(z)^2 \pi} \exp \left[ \frac{-2(x^2 + y^2)}{w(z)^2} \right], \quad (2.3.7)
\]

where \( w(z) \) is the \( 1/e^2 \) radius of the beam varying along its propagation axis, \( z \), and is given by

\[
w(z) = \sqrt{w_0^2 \left( 1 + \left( \frac{\lambda z}{\pi w_0^2} \right)^2 \right)}, \quad (2.3.8)
\]

where, \( w_0 \) is the beam waist radius at the point of best focus. For a Gaussian beam,
\[ \int_{-\infty}^{\infty} I(x \pm \Delta x/2, y, z) \, dx \, dy = 1 \text{ for any z}. \] Additionally, it is assumed that the integration bounds in \( z \) are equal as \( s_1 = s_2 = s \). With these assumptions, Eq. 2.3.6 is rewritten by bringing the local intensity into each line integral as

\[
\Delta \phi = \frac{2\pi}{\lambda} \left[ \int \int_{-\infty}^{\infty} \int_{s} I \left( x - \frac{\Delta x}{2}, y, z \right) n(x, y, z) \, dz \, dx \, dy - \right. \\
\left. \int \int_{-\infty}^{\infty} \int_{s} I \left( x + \frac{\Delta x}{2}, y, z \right) n(x, y, z) \, dz \, dx \, dy \right].
\] (2.3.9)

Next, the Gladstone-Dale relation, \( n = K \rho + 1 \), is used to relate the local index of refraction to the local density. Here, \( K \) is the Gladstone-Dale constant and \( \rho \) is the local density. Inserting the Gladstone-Dale relation into Eq. 2.3.9 and dividing by the beam intraspacing yields

\[
\frac{\Delta \phi}{\Delta x} = \frac{2\pi K}{\Delta x \lambda} \left[ \int \int_{-\infty}^{\infty} \int_{s} I \left( x - \frac{\Delta x}{2}, y, z \right) \rho(x, y, z) \, dz \, dx \, dy - \right. \\
\left. \int \int_{-\infty}^{\infty} \int_{s} I \left( x + \frac{\Delta x}{2}, y, z \right) \rho(x, y, z) \, dz \, dx \, dy \right] \\
= \frac{2\pi K}{\Delta x \lambda} \left[ \int \int_{-\infty}^{\infty} \int_{s} I \left( x - \frac{\Delta x}{2}, y, z \right) \rho(x, y, z) \, dz \, dx \, dy - \right. \\
\left. I \left( x + \frac{\Delta x}{2}, y, z \right) \rho(x, y, z) \right] \, dz \, dx \, dy \\
= \frac{2\pi K}{\Delta x \lambda} \left[ \int \int_{-\infty}^{\infty} \int_{s} \rho(x, y, z) \left[ I \left( x - \frac{\Delta x}{2}, y, z \right) - \right. \\
\left. I \left( x + \frac{\Delta x}{2}, y, z \right) \right] \, dz \, dx \, dy. \right] (2.3.10)

In the following sections, the second and third forms of Eq. 2.3.10 will be used to calculate \( \rho(x, y, z) \) in frequency space, given \( I(x, y, z) \) and some knowledge of the flow field.
2.4 Relation of Discrete Phase Change to Differential Phase Change via Transfer Function

In this section, the discrete phase change measured by FLDI will be related to a hypothetical, idealized differential FLDI response via transfer functions for the purpose of finding an expression for the density spectrum. Similar to Refs. [75] and [76], to model this hypothetical, idealized FLDI, the separation distance between the beams is reduced to a small value as

$$\frac{\partial \phi}{\partial x} = \lim_{\Delta x \to 0} \frac{\Delta \phi}{\Delta x}. \quad (2.4.1)$$

Substituting the first line of Eq. 2.3.10 into equation Eq. 2.4.1 for $\frac{\Delta \phi}{\Delta x}$, we get

$$\frac{\partial \phi}{\partial x} = \lim_{\Delta x \to 0} \frac{\Delta \phi}{\Delta x} \left[ \frac{2\pi K}{\lambda} \int \int_{s} I \left( x, y, z \right) \rho(x, y, z) \, dz \, dx \, dy - \int \int_{s} I \left( x + \frac{\Delta x}{2}, y, z \right) \rho(x, y, z) \, dz \, dx \, dy \right], \quad (2.4.2)$$

which reduces to

$$\frac{\partial \phi}{\partial x} = \frac{2\pi K}{\lambda} \int \int_{s} I(x, y, z) \left[ \lim_{\Delta x \to 0} \frac{\rho(x + \Delta x/2, y, z) - \rho(x - \Delta x/2, y, z)}{\Delta x} \right] \, dz \, dx \, dy \right] \quad (2.4.3)$$

$$= \frac{2\pi K}{\lambda} \int \int_{s} I(x, y, z) \frac{\partial \rho}{\partial x} \, dz \, dx \, dy.]$$
Now, the nature of an idealized FLDI instrument is defined by asserting that $\partial \rho / \partial x$ is evaluated at the instrument’s focus as,

$$\frac{\partial \rho(x, y, z)}{\partial x} \equiv \frac{\partial \rho(x, y, z)}{\partial x} \bigg|_{x, y, z = 0} = \frac{\partial \rho}{\partial x},$$

so it is no longer a function of space, and we note that $\int \int \int_{-\infty}^{\infty} I(x, y, z) \, dx \, dy = 1$. We can then write

$$\frac{\partial \phi}{\partial x} = \frac{2\pi K}{\lambda} \frac{\partial \rho}{\partial x} \left[ \int \int_{-\infty}^{\infty} I(x, y, z) \, dz \, dx \, dy \right] = \frac{2\pi K}{\lambda} \frac{\partial \rho}{\partial x} \int \int_{s} dz. \quad \text{(2.4.5)}$$

The path integration in Eq. 2.4.5 introduces a length scale over which the FLDI response is averaged. Here, the integration length can be approximated to be equal to the characteristic length of the FLDI probe volume, $L_p$,

$$\frac{\partial \phi}{\partial x} = \frac{2\pi K L_p}{\lambda} \frac{\partial \rho}{\partial x}. \quad \text{(2.4.6)}$$

A spectral analysis of the results is typically of interest, so we solve for $\frac{\partial \rho}{\partial x}$ and take the spatial Fourier transform of Eq. 2.4.6 as

$$\mathcal{F} \left\{ \frac{\partial \rho}{\partial x} \right\} = \frac{\lambda}{2\pi K L_p} \mathcal{F} \left\{ \frac{\partial \phi}{\partial x} \right\}. \quad \text{(2.4.7)}$$

The Fourier transform of the density is computed using the derivative property of the Fourier transform ($\mathcal{F} \{ \partial \rho / \partial x \} = i\kappa \mathcal{F} \{ \rho \}$), transforming from physical space to wavenumber ($\kappa$) space as

$$\mathcal{F} \{ \rho \} = P(\kappa) = \frac{\lambda}{2\pi i \kappa K L_p} \mathcal{F} \left\{ \frac{\partial \phi}{\partial x} \right\}. \quad \text{(2.4.8)}$$
The system transfer function of the FLDI instrument is defined in Eq. 2.4.9 as the ratio of the measured instrument output at the detector to the expected instrument output of an ideal FLDI instrument confined to a specification of our choosing within the framework presented above.

\[ H(\kappa) \equiv \frac{\left[ \frac{\Delta \phi}{\Delta x} \right]_{measured}}{\left[ \frac{\partial \phi}{\partial x} \right]_{ideal}}. \] (2.4.9)

The definition of the transfer function, \( H(\kappa) \), is used to relate the output of the instrument to the first derivative of the phase field. Solving for the derivative of the phase change in Eq. 2.4.9, we can make a substitution into Eq. 2.4.8 to obtain a relationship in wavenumber space between the measured fluctuations in phase to the actual density fluctuations as

\[ \mathcal{F}\{\rho\} = P(\kappa) = \frac{\lambda}{2\pi i \kappa KL_p \Delta x} \mathcal{F}\{\Delta \phi\} = \frac{\lambda}{2\pi i \kappa KL_p \Delta x} \frac{\Phi(\kappa)}{H(\kappa)}, \] (2.4.10)

where \( \Phi(\kappa) = \mathcal{F}\{\Delta \phi\} \). In the following sections, a model of the flow field will be assumed to determine \( H \). It is noted that relating the Fourier transform of density to the phase change in this manner follows Ref. [75] and Ref. [76].

### 2.5 Derivation of Transfer Functions

In this section, the transfer functions introduced by Ref. [75] and Ref. [76] will be re-derived with the framework described in the previous sections. Then, new transfer functions will be introduced that attempt to capture more general flow disturbances, namely isotropic turbulence. To first re-derive the functions in Ref. [75] and Ref. [76], we assume a sinusoidal disturbance in \( x \), uniform in \( y \), and infinitesimally thin in \( z \).
at $z = 0$ of the form

$$\rho = \rho(x, y, z) = C \sin(\kappa x + \phi_x)\delta(z), \quad (2.5.1)$$

where $\phi_x$ is an arbitrary phase shift along the x-direction, $C$ is an arbitrary constant, and $\delta(z)$ is the Dirac delta. Substituting the chosen form of the disturbance into Eq. 2.3.10 allows for the evaluation of the line integral as

$$\frac{\Delta \phi}{\Delta x} = \frac{2\pi KC}{\kappa \Delta x} \left[ \int_{\int_{-\infty}^{\infty} \sin(\kappa x + \phi_x) \left( I \left( x - \frac{\Delta x}{2}, y \right) - I \left( x + \frac{\Delta x}{2}, y \right) \right) dx\, dy \right]$$

$$= \frac{2\pi KC}{\kappa \Delta x} \frac{2}{\sin \left( \frac{\kappa \Delta x}{2} \right)} \exp \left( -\frac{w^2 \kappa^2}{8} \right) \cos(\phi_x). \quad (2.5.2)$$

The integration in Eq. 2.5.2 is similar to the sine transform of a Gaussian, so it is readily computed analytically, as is similarly done elsewhere in this section. To evaluate the transfer function $H(\kappa)$ for this disturbance, we must first evaluate $\frac{\partial \phi}{\partial x}$. Substituting Eq. 2.5.1 into Eq. 2.4.5 results in

$$\frac{\partial \phi}{\partial x} = \frac{2\pi K}{\lambda} \frac{\partial \rho}{\partial x} \int_{s} dz = \frac{2\pi KC}{\lambda} \kappa \cos(\kappa x + \phi_x)\delta(z) \bigg|_{x, y, z = 0} = \frac{2\pi KC}{\lambda} \kappa \cos(\phi_x), \quad (2.5.3)$$

noting that $\int_{s} dz = 1$ is chosen to represent the relevant integration length considered with the Dirac delta. The ratio of Eq. 2.5.2 to Eq. 2.5.3 is the transfer function

$$H(\kappa) = \frac{2}{\kappa \Delta x} \sin \left( \frac{\kappa \Delta x}{2} \right) \exp \left( -\frac{w^2 \kappa^2}{8} \right), \quad (2.5.4)$$

which is equivalent to Eq. 18 in Schmidt and Shepherd [76]. In that work, they formulate their Eq. 18 by combining two separately derived transfer functions, one
that accounts for the finite-differencing effects of FLDI, $H_s = 2 \sin(\kappa \Delta x/2)/(\kappa \Delta x)$ (their Eq. 17), and the effects of beam size at best focus, $H_{w,0} = \exp(-w_0^2\kappa^2/8)$ (their Eq. 15). Reproducing the result in Schmidt and Shepherd [76] brings confidence to the methodology of deriving transfer functions outlined in this work.

Next, a disturbance field over a finite domain of the form

$$\rho(x, y, z) = \begin{cases} 
C \sin(\kappa x + \phi_x) & -L \leq z \leq L \\
0 & \text{otherwise},
\end{cases} \quad (2.5.5)$$

is considered (Fig. 2.4a). Substituting the chosen form of the disturbance into Eq. 2.3.10 yields

$$\frac{\Delta \phi}{\Delta x} = \frac{2\pi KC}{\Delta x \lambda} \int \int_{-\infty}^{\infty} \int_{-L}^{L} \sin(\kappa x + \phi_x) \left[ I \left( x - \frac{\Delta x}{2}, y, z \right) - I \left( x + \frac{\Delta x}{2}, y, z \right) \right] dz \, dx \, dy$$

$$= \frac{2\pi KC}{\Delta x \lambda} 2 \sin \left( \frac{\kappa \Delta x}{2} \right) \cos(\phi_x) \int_{-L}^{L} \exp \left( -\frac{w(z)^2\kappa^2}{8} \right) dz$$

$$= \frac{2\pi KC}{\Delta x \lambda} 2 \sin \left( \frac{\kappa \Delta x}{2} \right) \cos(\phi_x) \frac{2\sqrt{2\pi}w_0\kappa^2}{\kappa \lambda} \exp \left( -\frac{w_0^2\kappa^2}{8} \right) \text{erf} \left[ \frac{L\kappa \lambda}{2\sqrt{2\pi}w_0} \right]. \quad (2.5.6)$$

Substituting Eq. 2.5.5 into Eq. 2.4.5 results in

$$\frac{\partial \phi}{\partial x} = \frac{2\pi KC}{\lambda} \int dz = \frac{2\pi KC}{\lambda} \kappa \cos(\kappa x + \phi_x) \bigg|_{x, y, z = 0}^{L}$$

$$= \frac{2\pi KC}{\lambda} 2L \kappa \cos(\phi_x), \quad (2.5.7)$$

noting that $L$ is chosen as the bound on the path integral to represent the finite length
of the disturbance field. The ratio of Eq. 2.5.6 to Eq. 2.5.7 is the transfer function

\[ H(\kappa) = \frac{2 \sqrt{2} \pi^{3/2} w_0}{\kappa^2 \lambda \Delta x L} \sin \left( \frac{\kappa \Delta x}{2} \right) \exp \left( -\frac{w_0^2 \kappa^2}{8} \right) \text{erf} \left[ \frac{L \kappa \lambda}{2 \sqrt{2} \pi w_0} \right], \]  

(2.5.8)

which is similar to a combination of Eqs. 16 and 17 in Schmidt and Shepherd [76]. Eq. 2.5.8 may be used as a transfer function for disturbances within a wind tunnel with walls from \(-L\) to \(L\). However, assuming a disturbance has the structure of Eq. 2.5.5 may not be the best representation of a real flow field as \(L\) becomes large relative to \(1/\kappa\).

Furthermore, when taking the ratio of Eq. 2.5.6 to Eq. 2.5.7 it is arbitrary to choose the integration limits of the idealized FLDI system as \(\pm L\) in Eq. 2.5.7 which results in an \(L\) in the denominator of Eq. 2.5.8. That is, for a given disturbance field, we can relate the phase response of the actual FLDI system (represented by Eq. 2.5.6) to the phase response of an idealized FLDI system having an integration length of one’s choosing. A convenient choice would be \(-L_P\) to \(L_P\), noting that \(L_P\) is the characteristic length of the FLDI probe volume in Eq. 2.4.10. Eq. 2.5.8 then becomes

\[ H(\kappa) = \frac{2 \sqrt{2} \pi^{3/2} w_0}{\kappa^2 \lambda \Delta x L_P} \sin \left( \frac{\kappa \Delta x}{2} \right) \exp \left( -\frac{w_0^2 \kappa^2}{8} \right) \text{erf} \left[ \frac{L \kappa \lambda}{2 \sqrt{2} \pi w_0} \right]. \]  

(2.5.9)

Importantly, setting the integration length to be the characteristic length of the FLDI instrument in Eq. 2.5.7 eliminates the need to characterize \(L_P\), as it cancels out in Eq. 2.4.10 when it is applied to reduce FLDI data.

Disturbances of increasingly complex form will now be introduced. First, to model isotropic turbulence with disturbances in \(x\) and \(y\) at the best focus of the FLDI
system, the density is assumed to take the form

\[ \rho = \rho(x, y, z) = C \sin(\kappa x + \phi_x) \sin(\kappa y + \phi_y) \delta(z). \]  \hspace{1cm} (2.5.10)

Substituting the chosen form of the disturbance into Eq. 2.3.10 yields

\[ \frac{\Delta \phi}{\Delta x} = \frac{2\pi KC}{\Delta x \lambda} \int \int_{-\infty}^{\infty} \sin(\kappa x + \phi_x) \sin(\kappa y + \phi_y) \left[ I \left( x - \frac{\Delta x}{2}, y, z \right) - I \left( x + \frac{\Delta x}{2}, y, z \right) \right] \, dx \, dy \]

\[ = \frac{2\pi KC}{\Delta x \lambda} 2 \sin \left( \frac{\kappa \Delta x}{2} \right) \exp \left( -\frac{w_0^2 \kappa^2}{4} \right) \cos(\phi_x) \sin(\phi_y). \]  \hspace{1cm} (2.5.11)

Substituting Eq. 2.5.10 into Eq. 2.4.5 results in

\[ \frac{\partial \phi}{\partial x} = \frac{2\pi K}{\lambda} \frac{\partial \rho}{\partial x} \int_s \, dz \]

\[ = \frac{2\pi KC}{\lambda} \kappa \cos(\kappa x + \phi_x) \sin(\kappa y + \phi_y) \delta(z) \bigg|_{x,y,z=0} \]  \hspace{1cm} (2.5.12)

The ratio of Eq. 2.5.11 to Eq. 2.5.12 is the transfer function

\[ H(\kappa) = \frac{2}{\kappa \Delta x} \sin \left( \frac{\kappa \Delta x}{2} \right) \exp \left( -\frac{w_0^2 \kappa^2}{4} \right) \]  \hspace{1cm} (2.5.13)

noting that the only change between Eq. 2.5.13 and Eq. 2.5.4 is the factor of two in the exponential.

Next, consider an isotropic disturbance field (see Fig. 2.4b) of the form

\[ \rho(x, y, z) = \begin{cases} 
C \sin(\kappa x + \phi_x) \sin(\kappa y + \phi_y) & -L \leq z \leq L \\
0 & \text{otherwise},
\end{cases} \]  \hspace{1cm} (2.5.14)
which, following the above process, yields

\[
\frac{\Delta \phi}{\Delta x} = \frac{2\pi KC}{\Delta x \lambda} \int \int_{-\infty}^{\infty} \int_{-L}^{L} \sin(\kappa x + \phi_x) \sin(\kappa y + \phi_y) \left[ I \left( x - \frac{\Delta x}{2}, y, z \right) - I \left( x + \frac{\Delta x}{2}, y, z \right) \right] dz \, dx \, dy
\]

\[
= \frac{2\pi KC}{\Delta x \lambda} 2 \sin \left( \frac{\kappa \Delta x}{2} \right) \cos(\phi_x) \sin(\phi_y) \int_{-L}^{L} \exp \left( - \frac{w(z)^2 \kappa^2}{4} \right) dz \tag{2.5.15}
\]

\[
= \frac{2\pi KC}{\Delta x \lambda} 2 \sin \left( \frac{\kappa \Delta x}{2} \right) \cos(\phi_x) \sin(\phi_y) \frac{2\pi^{3/2}w_0}{\kappa \lambda} \times \exp \left( - \frac{w_0^2 \kappa^2}{4} \right) \text{erf} \left[ \frac{L \kappa \lambda}{2\pi w_0} \right].
\]

Substituting Eq. 2.5.14 into Eq. 2.4.5 results in

\[
\frac{\partial \phi}{\partial x} = \frac{2\pi K}{\lambda} \frac{\partial \rho}{\partial x} \int_{s} dz = \frac{2\pi KC}{\lambda} \kappa \cos(\kappa x + \phi_x) \sin(\kappa y + \phi_y) \bigg|_{x,y,z=0} \int_{-L}^{L} dz \tag{2.5.16}
\]

and taking the ratio of Eq. 2.5.15 to Eq. 2.5.16 gives the transfer function

\[
H(\kappa) = \frac{2\pi^{3/2}w_0}{\kappa^2 \lambda \Delta x L} \sin \left( \frac{\kappa \Delta x}{2} \right) \exp \left( - \frac{w_0^2 \kappa^2}{4} \right) \text{erf} \left[ \frac{L \kappa \lambda}{2\pi w_0} \right]. \tag{2.5.17}
\]

Finally, in terms of considering disturbances within a fixed boundary \( L \), we consider a three-dimensional isotropic disturbance field (Fig. 2.4c) of the form

\[
\rho(x, y, z) = \begin{cases} 
C \sin(\kappa x + \phi_x) \sin(\kappa y + \phi_y) \sin(\kappa z + \phi_z) & -L \leq z \leq L \\
0 & \text{otherwise},
\end{cases} \tag{2.5.18}
\]
Figure 2.4: Representation of density disturbance fields of the form (a) \( \rho = \rho(x, y, z) = \sin(\kappa x + \phi_x) \), (b) \( \rho = \rho(x, y, z) = \sin(\kappa x + \phi_x) \sin(\kappa y + \phi_y) \), and (c) \( \rho = \rho(x, y, z) = \sin(\kappa x + \phi_x) \sin(\kappa y + \phi_y) \sin(\kappa z + \phi_z) \). Due to the additional complexity of the disturbance field, slices of the field are shown in (c).

Substituting this disturbance into the phase change relation yields

\[
\frac{\Delta \phi}{\Delta x} = \frac{2\pi KC}{\Delta x \lambda} \int \int \int \sin(\kappa x + \phi_x) \sin(\kappa y + \phi_y) \sin(\kappa z + \phi_z) \times \\
\left[ I \left( x - \frac{\Delta x}{2}, y, z \right) - I \left( x + \frac{\Delta x}{2}, y, z \right) \right] dz \, dx \, dy
\]

\[
= \frac{2\pi KC}{\Delta x \lambda} 2 \sin \left( \frac{\kappa \Delta x}{2} \right) \cos(\phi_x) \sin(\phi_y) \int_{-L}^{L} \sin(\kappa z + \phi_z) \times \\
\exp \left( -\frac{w(z)^2 \kappa^2}{4} \right) \right) dz
\]

\[
= \frac{2\pi KC}{\Delta x \lambda} 2 \sin \left( \frac{\kappa \Delta x}{2} \right) \cos(\phi_x) \sin(\phi_y) \sin(\phi_z) \frac{i \pi^{3/2} w_0}{\kappa \lambda} \times \\
\exp \left[ -\frac{w_0^2}{4} \left( \kappa^2 + \frac{4\pi^2}{\lambda^2} \right) \right] \times \left[ \text{erfi} \left( \frac{\pi w_0}{\lambda} - \frac{i L \kappa \lambda}{2\pi w_0} \right) - \\
\text{erfi} \left( \frac{\pi w_0}{\lambda} + \frac{i L \kappa \lambda}{2\pi w_0} \right) \right],
\]

noting that \( \text{erf} \) and \( \text{erfi} \) are the error function and imaginary error function, respectively. Following the same procedure that was used to obtain Eq. 2.5.17, it is found
that
\begin{equation}
H(\kappa) = \frac{i\pi^{3/2}w_0}{\kappa^2\lambda\Delta xL} \exp \left[ -\frac{w_0^2}{4} \left( \frac{\kappa^2 + 4\pi^2}{\lambda^2} \right) \right] \sin \left( \frac{\kappa\Delta x}{2} \right) \times \exp \left[ \frac{\pi w_0}{\lambda} - \frac{iL\kappa\lambda}{2\pi w_0} \right] \right] - \text{erfi} \left( \frac{\pi w_0}{\lambda} + \frac{iL\kappa\lambda}{2\pi w_0} \right) \right].
\end{equation}

This expression is simplified using the identities \(i \times \text{erfi}(z) = \text{erf}(i \times z), \text{erf}(-z) = -\text{erf}(z),\) and \(2\Re[\text{erf}(x + i \times y)] = \text{erf}(x + i \times y) + \text{erf}(x - i \times y)\) with \(x = \frac{L\kappa\lambda}{2\pi w_0}\) and \(y = \frac{\pi w_0}{\lambda}\). With this, Eq. 2.5.20 becomes
\begin{equation}
H(\kappa) = \frac{\pi^{3/2}w_0}{\kappa^2\lambda\Delta xL} \exp \left[ -\frac{w_0^2}{4} \left( \frac{\kappa^2 + 4\pi^2}{\lambda^2} \right) \right] \sin \left( \frac{\kappa\Delta x}{2} \right) \times \exp \left[ 2\Re \left[ \frac{i\pi w_0}{\lambda} + \frac{L\kappa\lambda}{2\pi w_0} \right] \right] \right].
\end{equation}

Note that the \(L\) in the denominator in Eqs. 2.5.17 or 2.5.21 could be written as \(L_P\), as shown in Eq. 2.5.9.

As an alternative to considering disturbances within a fixed boundary, \(L\), we can assume a sinusoidal disturbance in \(x\), with a Gaussian width \(\sigma\) as
\begin{equation}
\rho = \rho(x,y,z) = C \sin(\kappa x + \phi_x) \exp \left( -\frac{y^2 + z^2}{\sigma^2} \right) .
\end{equation}

This model may be useful to determine the response of an FLDI system to a axisymmetric turbulent jet of width \(\sigma\) propagating in the \(x\) direction centered at \(y = z = 0\).
Inserting this form of the disturbance into the phase-change relation, we get

\[
\frac{\Delta \phi}{\Delta x} = 2\pi KC \frac{\Delta x}{\lambda} \int \int \int \sin(\kappa x + \phi_x) \exp \left(-\frac{y^2 + z^2}{\sigma^2}\right) \times \left[I \left(x - \frac{\Delta x}{2}, y, z\right) - I \left(x + \frac{\Delta x}{2}, y, z\right) \right] dz \, dx \, dy
\]

\[
= 2\pi KC \frac{\Delta x}{\lambda} 2 \sin \left(\frac{\kappa \Delta x}{2}\right) \cos(\phi_x) \int_{-\infty}^{\infty} \frac{1}{\sqrt{\frac{w(z)^2}{\sigma^2} + 1}} \times \exp \left(-\frac{w(z)^2 \kappa^2}{8}\right) \exp \left(-\frac{-z^2}{\sigma^2}\right) dz,
\]

\[
= 2\pi KC \frac{\Delta x}{\lambda} 2 \sin \left(\frac{\kappa \Delta x}{2}\right) \cos(\phi_x) \int_{-\infty}^{\infty} \exp \left(-\frac{w(z)^2 \kappa^2}{8}\right) \exp \left(-\frac{-z^2}{\sigma^2}\right) dz,
\]

\[
= 2\pi KC \frac{\Delta x}{\lambda} 2 \sin \left(\frac{\kappa \Delta x}{2}\right) \cos(\phi_x) \frac{4\pi^{3/2}}{\sqrt{\frac{\kappa^2 \lambda^2}{2w_0^2} + \frac{4\pi^2}{\sigma^2}}}.
\]

\(\text{(2.5.23)}\)

To make the integration in \(z\) on the second line of Eq. \(2.5.23\) tractable, we assume that the beam waist is much smaller than the jet width, \(\sigma >> w(z)\). Inserting Eq. \(2.5.22\) into Eq. \(2.4.5\) results in

\[
\frac{\partial \phi}{\partial x} = \frac{2\pi K}{\lambda} \frac{\partial \rho}{\partial x} \int_{s} dz = \frac{2\pi KC}{\lambda} \kappa \cos(\kappa x + \phi_x) \exp \left(-\frac{y^2 + z^2}{\sigma^2}\right) \bigg|_{x,y,z=0}^{x,y,z=\sigma} dz
\]

\[
= \frac{2\pi KC}{\lambda} -2\sigma \kappa \cos(\phi_x),
\]

\(\text{(2.5.24)}\)

and taking the ratio of Eq. \(2.5.22\) to Eq. \(2.5.24\) the transfer function is

\[
H(\kappa) = \frac{2\pi^{3/2} \sin \left(\frac{\kappa \Delta x}{2}\right) \exp \left[-\frac{1}{8}\kappa^2 w_0^2\right]}{\sigma \kappa \Delta x \sqrt{\frac{\kappa^2 \lambda^2}{2w_0^2} + \frac{4\pi^2}{\sigma^2}}}.
\]

\(\text{(2.5.25)}\)

For an increasingly complex disturbance in \(x\) and \(y\) with a Gaussian width \(\sigma\) as

\[
\rho = \rho(x, y, z) = C \sin(\kappa x + \phi_x) \sin(\kappa y + \phi_y) \exp \left(-\frac{y^2 + z^2}{\sigma^2}\right),
\]

\(\text{(2.5.26)}\)
yields
\[ H(\kappa) = \frac{2\pi^{3/2} \sin \left( \frac{\kappa \Delta x}{2} \right) \exp \left[ -\frac{1}{4} \kappa^2 w_0^2 \right]}{\sigma \kappa \Delta x \sqrt{\frac{\kappa^2 \lambda^2}{2w_0^2} + \frac{4\pi^2}{\sigma^2}}} , \]  
(2.5.27)

following the above procedure and assumptions. Finally, assuming a disturbance of the form

\[ \rho = \rho(x, y, z) = C \sin(\kappa x + \phi_x) \sin(\kappa y + \phi_y) \sin(\kappa z + \phi_z) \exp\left( -\frac{y^2 + z^2}{\sigma^2} \right) , \]  
(2.5.28)

yields
\[ H(\kappa) = \frac{2\pi^{3/2} \sin \left( \frac{\kappa \Delta x}{2} \right) \exp \left[ -\frac{1}{4} w_0^2 \kappa^2 \left( 1 + \frac{4\pi^2 \sigma^2}{4\pi^2 w_0^2 + \kappa^2 \lambda^2 \sigma^2} \right) \right]}{\kappa \Delta x \sigma \sqrt{\kappa^2 \lambda^2 / w_0^2 + 4\pi^2 / \sigma^2}} . \]  
(2.5.29)

For each of the transfer functions pertaining to the axisymmetric jet, Eqs. 2.5.25, 2.5.27 and 2.5.29, the FLDI user must acknowledge that one is integrating and averaging through the spanwise structure of the jet in the z direction.

2.6 Run Condition Calculation and Stability Analysis

For each experiment performed, the nozzle reservoir conditions are used to estimate the freestream run conditions. The thermodynamic state of the test gas in the nozzle reservoir is determined using the shock tube pressure, \( P_1 \), and the measured incident shock speed, \( U_s \). Using Cantera [78] with the Shock and Detonation Toolbox [79], the gas is assumed to isentropically expand to the reservoir pressure, \( P_R \), accounting for the weak expansion or compression waves that are reflected between the contact surface and the shock tube end wall [30]. The calculated nozzle reservoir conditions are inputted into the University of Minnesota Nozzle Code to determine the freestream conditions at the exit of the contoured nozzle [80-83]. A representative grid used in the
nozzle simulation of the experiments performed at California Institute of Technology’s T5 facility is shown in Fig. 2.5. The grid, generated in Eilmer [84], is clustered near the throat and the wall of the nozzle to adequately resolve the complex flow in these regions. The freestream conditions are chosen to be an areal average of the simulation output at the approximate distance from the throat to the location of the cone’s nose tip.

![Figure 2.5: Grid used in the nozzle simulations for experiments performed at Caltech’s T5 facility. The red rectangle denotes the area in which the simulation output was averaged to obtain the freestream conditions.](image)

The Stability and Transition Analysis for Hypersonic Boundary Layers (STABL) software suite was used to perform stability analysis for the experiments described in this work. STABL uses NASA’s data-parallel lower-upper relaxation (DPLR) method as described by Wright et al. [80], Johnson [82], and Johnson et al. [85] to solve the mean flow equations. First, the method of normal modes is applied to the reacting Navier-Stokes equations, where it is assumed that the boundary layer is quasi-parallel, the gas is in thermo-chemical non-equilibrium, and the disturbances have the form

\[ q'(s, z, t) = \hat{q}(y) \exp(i(\alpha s + \beta z - \omega t)), \]  

(2.6.1)

where \( q' \) is a disturbance at a position along the generator of the cone \( s \), azimuthal position \( z \), and time \( t \). The amplitude of the disturbance is considered to be only a function of the wall-normal distance, \( \hat{q} = \hat{q}(y) \). The stream-wise wave number
is $\alpha$, the azimuthal wave number is $\beta$, and the frequency is $\omega$. The spatial linear stability problem is analyzed where the frequency ($\omega$) is real, and the wave numbers are complex ($\alpha = \alpha_r + \alpha_i$). The linear stability calculation results are then used as initial values for the parabolized stability equation (PSE) analysis, which is used to account for the non-parallel nature of the boundary layer. The procedure for the PSE analysis is found in Johnson [82].

A parabolized stability equation solver, PSE-Chem, is used to solve the parabolized stability equations and compute linear stability diagrams and amplification curves. PSE-Chem allows for the simulation of finite-rate chemistry effects and transition-vibrational energy exchange to predict the growth rate of naturally-occurring disturbances in hypersonic flows [86]. The amplification curves are computed at the flow diagnostic measurement position along the cone. The amplification factor, $N$, growth rate, $\alpha$, and disturbance kinetic energy, $E$, are computed as

$$N = \int \sigma ds,$$  \hspace{1cm} (2.6.2a)

$$\sigma = -\text{Im}(\alpha) + \frac{1}{2E} \frac{dE}{ds},$$  \hspace{1cm} (2.6.2b)

$$E = \int_{\Omega} \bar{\rho} \left( |\hat{u}|^2 + |\hat{v}|^2 + |\hat{w}|^2 \right) dV,$$  \hspace{1cm} (2.6.2c)

where $\alpha$ is the wavenumber along a generator of the cone.

The grid generation module of the STABL software suite was used to generate specific grids for the experiments analyzed in this research. For the analysis of models with blunt nose tips (used in the experiments performed at California Institute of Technology’s T5 reflected shock tunnel), the grid tailoring routine was employed within STABL’s software suite to capture a shock-fitted grid. First, an intentionally oversized initial grid was used to compute the initial mean flow solution to ensure the
shock was completely captured. Using this initial grid, the mean flow was resolved with particular attention paid to the blunt nose tip to ensure the local residual is low enough to discern the shock in this region. After resolving the flow around the cone’s blunt nose tip, the solution was “frozen” in this area and the mean flow along the rest of the model was converged. A different set of Courant–Friedrichs–Lewy (CFL) values were used for this iteration of the mean-flow solution. The mean flow is deemed resolved once the residual throughout the cone is observed to reach approximately 1e-12.

Next, the initial, resolved mean flow solution was post-processed. As part of the grid tailoring process, STABL adjusts the upper grid definition and body-normal spacing to better fit the grid to the shock. During this step, particular attention was paid to ensure the $y^+$ wall value was less than 1 along the length of the cone, indicating a sufficient resolution of the boundary-layer profile. In a process similar to the one used with the initial grid, the mean flow analysis was rerun with the tailored grid to produce a higher quality mean flow solution for input into the stability analysis. The mean flow analyses for all experiments were performed on a high-performance computing cluster at Stevens Institute of Technology. Each mean-flow simulation took approximately 6 minutes running on 4 nodes with 16 tasks per node using a total of 64 processors. Adjustments were made to the grid, the maximum number of iterative steps, and the CFL ramp to achieve residual values of approximately 1e-12.

The stability analysis of the flow was performed using the parabolized stability equation solver within STABL. The test matrix was automatically generated within the PSE-Chem Analysis module. As the freestream Mach number was greater than 5 for these experiments, the two-dimensional second-mode instability was assumed to be the most unstable disturbance and non-zero azimuthal wavenumbers ($\beta$) representing oblique disturbances were not considered in the analysis.
Chapter 3
Benchtop Experiments Performed using FLDI

In this chapter, experiments performed in a laboratory setting using FLDI are discussed. The first experiment characterizes the response of the FLDI instrument to a controlled phase object. A cylindrical lens, serving as the uni-directional phase object, is placed in the beam path of the FLDI instrument, and vibrated using a speaker. The displacement of the lens is computed from an accelerometer attached to this lens and, with knowledge of the lens geometry, is converted to a phase change. A comparison between the phase change derived from the accelerometer signal to that measured by the FLDI instrument shows excellent agreement in both amplitude and frequency.

In the second experiment, a turbulent jet is probed by an FLDI instrument. The data-reduction techniques presented in Chapter 2 are applied and indicate that increasing the complexity of the transfer function has merit.

3.1 Focused Laser Differential Interferometer Response to a Controlled Phase Object

In this section, the response of the FLDI instrument is characterized using a controlled phase object. The instrument’s response to a changing optical path length is measured and compared to the vibrations measured by an accelerometer, serving as calibration for both amplitude and frequency.
3.1.1 Experimental Setup

For this experiment, a single-beam pair FLDI instrument was constructed using a Wollaston prism with a 1 arcminute splitting angle and a two-beam pair FLDI instrument was constructed by adding a Wollaston prism with a 20 arcminute splitting angle. The components of the two-beam pair FLDI setup are shown in Fig. 3.1a. For both the single beam-pair FLDI and two-beam pair FLDI setups, the beams were inter- and intraspaced in the \( x \) direction. For the single-beam pair FLDI setup, the beam intraspacing was 41 \( \mu m \). For the two-beam pair FLDI setup, the beam intraspacing was 41 \( \mu m \) and the beam interspacing was 699 \( \mu m \). A picture of the beams at the focus of the single-beam pair FLDI instrument is shown in Fig. 3.1b.

Figure 3.1: (a) Schematic of components used to generate a single-beam pair FLDI setup. (b) Picture of a single-beam pair FLDI instrument taken at the focus using a beam-profiling camera. The minor tick marks are at every 10 \( \mu m \) and the major tick marks are at every 100 \( \mu m \). A two-beam pair FLDI setup is developed by adding a quarter-wave plate and Wollaston prism bundle to the upbeam side of the FLDI instrument. \( C_4 \) represents the lens used as the controlled phase object in these experiments.

To measure the response of the FLDI setup to a changing optical path length, the apparatus pictured in Fig. 3.2 was constructed. The apparatus was mounted on a manually adjustable translation stage and consisted of a speaker, an accelerometer, and a diverging cylindrical lens as a phase object. The setup could be precisely
adjusted in the ±z-direction using the translation stage. The speaker was driven at a prescribed frequency by a Stanford Research Systems model DS345 function generator. The diverging lens was suspended by a compliant spring directly in front of the speaker. The radius of curvature of the lens was along the direction of beam separation. The lens was positioned such that it gently touched the speaker, and it was placed in the path of the FLDI beams. As the speaker was driven at the prescribed frequency, its vibrations oscillated the lens along the direction of beam separation, changing the individual lengths traversed by each FLDI beam. A PCB 352C34 accelerometer measured the apparatus's acceleration and was mounted directly to the lens, in-line with the direction of oscillation. The apparatus described above modeled a density disturbance field experienced by an FLDI instrument that is sinusoidal in \( x \), uniform in \( y \), and assumed to be infinitesimally thin in \( z \), of the form \( \rho = \rho(x, y, z) = \sin(\kappa x)\delta(z) \).

For the single-beam pair FLDI setup, a diverging cylindrical lens of -50 mm was used as the phase object. For the two-beam pair FLDI setup, weaker lenses of focal lengths -150 mm and -400 mm were used to reduce beam steering and distortion. Initially, the apparatus was placed such that the lens was at the focus of the FLDI beams, i.e., \( z = 0 \). As the speaker was vibrated at the driving frequency, a comparison was made between the frequency measured by the FLDI instrument and the accelerometer, and between the amplitude of the phase change as measured by the FLDI instrument and as calculated using the accelerometer.

### 3.1.2 Phase Change Due to Cylindrical Lens Displacement

Similar to Ceruzzi and Cadou [57], the optical path lengths (OPLs) of two FLDI beams as they pass through a lens of radius of curvature \( R \) is shown in Fig. 3.3. The phase difference experienced by the FLDI beam pairs of wavelength \( \lambda \) due to their
distinct optical paths is

\[ \Delta \phi = \frac{2\pi}{\lambda} \Delta OPL = \frac{2\pi}{\lambda} \left( (L_1 n_a + L_3 n_g) - (L_2 n_a + L_4 n_g) \right), \tag{3.1.1} \]

where \( \Delta OPL \) represents the difference in optical path lengths between the individual beam pairs, \( L \) is distance, \( n_g \) is the refractive index of the glass, and \( n_a \) is the refractive index of the ambient environment.

The lengths can be found in terms of the geometry of the optical setup as

\[ L_1^2 + (\Delta x/2 - x)^2 = R^2, \tag{3.1.2a} \]

\[ L_2^2 + (\Delta x/2 + x)^2 = R^2, \tag{3.1.2b} \]
Figure 3.3: Cylindrical lens of radius of curvature $R$ with FLDI beams (red) displaced by $\Delta x$. The center of the FLDI beams is displaced a distance $x$ from the origin.

\[ L_3 = R - L_1, \quad (3.1.2c) \]

\[ L_4 = R - L_2. \quad (3.1.2d) \]

Simplifying the above relations results in

\[ \Delta \phi = \frac{2\pi}{\lambda} \left[ (n_g - n_a) \left( \sqrt{R^2 - (\Delta x/2 - x)^2} - \sqrt{R^2 - (\Delta x/2 + x)^2} \right) \right], \quad (3.1.3) \]

which enables the calculation of phase change if the position, $x$, of the lens is known.

The position is determined with an accelerometer, which may be independently compared to the phase change from the FLDI signal as

\[ \Delta \phi = \sin^{-1} \left( \frac{V}{V_0} - 1 \right), \quad (3.1.4) \]

where $V_0$ is the voltage at the linear part of a fringe. It is obtained by averaging
the minimum and maximum voltage output of the FLDI instrument measured by a photodetector as the downbeam Wollaston prism is adjusted to translate the FLDI instrument through a fringe.

3.1.3 Results and Discussion

In this section, the phase change as measured by the FLDI instrument is compared to the phase change as measured by the accelerometer. Results from an experiment with a single beam-pair FLDI setup and a $f_L = -50$ mm focal length cylindrical diverging lens used as the phase object placed at the focus are presented first. Fig. 3.4a shows the acceleration, velocity, and position of the phase object as measured by the accelerometer. The acceleration, $a(t)$, is numerically integrated once in the time domain to obtain the velocity, $v(t)$, and again to obtain the position, $x(t)$. Alternatively, the position of the phase object can be obtained in frequency space by pre-multiplication of the fast Fourier transform (FFT) of the acceleration data as $F[a(t)]/(4\pi^2 f^2)$. Comparison of the two methods is presented in Fig. 3.4b. Moreover, the amplitude of the fundamental frequency in Fig. 3.4b at 50 Hz matches the observed amplitude in Fig. 3.4a.

The position, $x(t)$, of the lens, calculated from the accelerometer signal, is substituted into Eq. 3.1.3 to directly obtain the phase change, $\Delta \phi$, as measured by the accelerometer. The phase change as measured by the FLDI instrument is determined by substituting the voltage output of the photodetector, $V(t)$, into Eq. 3.1.4. Both of these measurements of phase change are presented in Fig. 3.5a. The spectrum of the phase change from the accelerometer data is obtained by substituting $|x(f)| = F[a(t)]/(4\pi^2 f^2)$ into Eq. 3.1.3. A comparison of the accelerometer-derived and FLDI-derived phase-change spectrum is presented in Fig. 3.5b, noting excellent agreement at the fundamental frequency (50 Hz) and a half-dozen harmonics.
Figure 3.4: (a) Acceleration, velocity and position of the phase object as measured by the accelerometer; velocity and position are obtained by numerical integration. (b) FFT of the position of the lens as computed by the numerical integration of accelerometer data (time domain) and as computed by the pre-multiplication of the FFT of the acceleration data, $\mathcal{F}[a(t)]/(4\pi^2 f^2)$, (frequency domain).

Figure 3.5: Phase change as measured by the accelerometer and the FLDI instrument as a function of (a) time and (b) frequency.

The phase object is next translated in the $z$-direction and comparisons are made between the response of the accelerometer and the FLDI instrument. On the left of Fig. 3.6 from top to bottom, pictures of the FLDI beam pair are presented at $z = 2.54$ mm, $5.08$ mm, and $7.62$ mm away from the beam’s focus. On the right of
Fig. 3.6, we present the phase change as measured by the accelerometer and the FLDI instrument at the corresponding positions. As the phase object is translated away from the focus of the FLDI instrument, the increasing $1/e^2$ beam radius results in beam overlap and signal attenuation of the FLDI instrument. For example, at ± 7.62 mm away from the focus, the FLDI instrument’s signal is reduced to approximately 25% of its signal at the focus. For the FLDI setup built for this experiment, this deviation from the focus represents less than 2% of the total span along which the FLDI instrument is sensitive to phase differences. Fig. 3.7 summarizes the signal attenuation experienced by the FLDI instrument as the phase object is moved away from the focus. A consistent reduction in the FLDI signal is observed at increasing distances away from the focus, although no trend with frequency is readily apparent.

In Fig. 3.8, results are presented for an experiment with a two beam-pair FLDI setup, with the two beams pairs identified as FLDI A and FLDI B. Controlled phase objects in the form of diverging cylindrical lenses with focal lengths of $f_L = -150$ mm and $f_L = -400$ mm were placed at the focus of the two beam-pair FLDI instrument and vibrated at a fundamental frequency of 50 Hz. The FFT shows excellent agreement between the accelerometer and the two FLDI signals in picking up the fundamental frequency and higher frequency resonances inherent to the apparatus. There is good agreement in the amplitude of the phase change between the two measurement methods noting reduced response with the lens of larger focal length (increased radius of curvature).

3.1.4 Conclusions

The FLDI instrument is characterized by comparing its response to a controlled phase object. The controlled phase object was a vibrating lens placed at the focus of the FLDI. Attached to this lens was an accelerometer from which displacement and,
Figure 3.6: Phase change as measured by an accelerometer and an FLDI instrument with an $f = -50$ mm diverging lens used as a phase object. (Left) From top to bottom: Pictures of FLDI beams taken at $z = 2.54$ mm, $z = 5.08$ mm, and $z = 7.62$ mm. Minor tick marks are at every 10 $\mu$m and major tick marks are at every 100 $\mu$m. (Right) Comparison of phase change as measured by the accelerometer and the FLDI instrument with the phase object positioned at the corresponding positions in $z$.

subsequently, phase change was determined. This accelerometer-derived phase change was found to be in excellent agreement with FLDI-derived phase change for both the
Figure 3.7: Ratio of the FLDI-derived phase change to accelerometer-derived phase-change with varying locations from the focus, $z$.

Figure 3.8: Comparison of accelerometer-derived phase change to two-beam pair FLDI-derived phase change at the focus. (a) $f_L = -150$ mm, and (b) $f_L = -400$ mm.

Off-focus measurements of phase change made with the FLDI were compared with the accelerometer acting as a control. The results show that the FLDI response is attenuated as the distance from the focus is increased.
3.2  FLDI Investigation of Turbulent Jet Spectra

In this section, an FLDI instrument is used to probe a turbulent jet. The transfer functions derived in Chapter 2 are applied in the reduction of FLDI data, and results are compared with the one-dimensional turbulence spectrum.

3.2.1  Experimental Setup

A schematic of the FLDI instrument constructed to probe the exit of a turbulent jet is shown in Fig. 3.9. Diffractive optical elements were used to generate the beam interspacing and Wollaston prisms having separation angles of 0.5 arcminutes, 1 arcminute, and 2 arcminutes were used for these experiments. A picture of the beam inter- and intraspace generated using a 2 arcminute Wollaston prism is presented in Fig. 3.10. The beam interspacing was 1.64 mm, and the beam intraspace was 263 µm. The beam interspace did not change appreciably for the other Wollaston prisms used in this experimental campaign. The beam intraspace using the 1 arcminute Wollaston prism was 85 µm and the intraspace using the 0.5 arcminute Wollaston prism was 36 µm.

Figure 3.9: Schematic of FLDI setup used to probe the exit of a turbulent jet.
A round, sonic free-jet was used to generate the turbulent disturbance field being probed by the FLDI beams. The free-jet was generated in a laboratory setting. Compressed air was regulated to approximately 30 PSIG in the reservoir of a nozzle with an exit diameter of 3.7 mm. The nozzle was mounted on a platform that allowed for independent and precise adjustment in the $x$, $y$, and $z$ directions. For these experiments, the nozzle was positioned at the focus ($z = 0$), 43 mm ($x/D = 11.6$) away from the FLDI beam pairs.

### 3.2.2 Results and Discussion

Results from the experiments are presented in this section. First, a temporal correlation between the two closely spaced FLDI probes yields the dispersion relation, $\kappa = 2\pi f/c_p(f)$. The phase speed, $c_p(f)$, was determined following a procedure similar to the one described by Ceruzzi et al. [87]. Fig. 3.11 shows the dispersion relation fits for convective velocities measured with 0.5, 1, and 2 arcminute Wollaston prisms. For these experiments, an inverse tangent function provided the most natural fit to the discretely calculated convective velocities and resulted in a functional relationship of the data points. The dispersion relation demonstrates the dependency of the disturbance convective velocity to the frequency. That is, the disturbances propagating at higher convective velocities tend to fluctuate at higher frequencies. Fig. 3.11
Figure 3.11: Convective velocities and fits of dispersion relation for experiments with 0.5, 1, and 2 arcminute Wollaston prisms.

also shows the similarity between the dispersion relations generated using the three Wollaston prisms, and demonstrates the independence of the measured dispersion to the optical setup for this flow field.

Figure 3.12 shows the transfer functions for the experiment using a 0.5 arcminute Wollaston prism. For these experiments, the length scale in the transfer function (2L or σ) is relatively small, and thus, the transfer function modifies the spectrum most at relatively high wavenumbers. The transfer functions of three-dimensional disturbance fields (Eqs. 2.5.21 and 2.5.29) suffer from a steep drop in magnitude at a much earlier than expected value of κ_1η, and are not presented in this figure. Their poor behavior most likely stems from the assumption that the disturbances are perfectly correlated along the z direction. Although the disturbances in x and y are also not perfectly correlated, as the simpler one- and two-dimensional transfer functions assume, the ratio of the disturbance length-scale to the integration length is less problematic. That is, 1/κ is closer in length scale to the beam waist
Figure 3.12: Calculated transfer functions for an experiment with a 0.5 arcminute Wollaston prism. Here, $\kappa_1$ refers to the wavenumber of the streamwise fluctuations.

$w(z)$ (for $x$ and $y$ integration) than it is to $L$ (for $z$ integration). This is an apparent limitation of the present approach where it is assumed disturbances are isotropic and perfectly correlated in all directions.

Here, the one-dimensional energy spectra of the density fluctuations are defined to be the one-dimensional Fourier transform of the autocorrelation function, $R_{11\rho}(x)$, analogous to the energy spectrum formed from velocity fluctuations in Pope [88]. Unfortunately, the autocorrelation function in x-space is not directly available from FLDI data because of the presence of dispersion (Fig. 3.11); that is, Taylor’s hypothesis may not be applicable in the present jet experiments. Additionally, the FLDI spectra need to be analyzed in wavenumber space so that the transfer functions may be applied. The Wiener-Khinchin Theorem provides a direct relationship between the Fourier transform of the density fluctuations to the autocorrelation function, and
is used to write the energy spectra of the density fluctuations as

\[ E_{11\rho}(\kappa_1) = \mathcal{F}\{R_{11\rho}(x)\} = \mathcal{F}\{\rho(x)\} \mathcal{F}\{\rho(x)\}^* = P(\kappa)P(\kappa)^*, \]  

(3.2.1)

where the * denotes the complex conjugate and \( P(\kappa) \) is the discrete Fourier spectrum of density fluctuations per Eq. 2.4.10 as calculated with the “fft” function in MATLAB. Note that the transfer function alters the energy spectra as \( E_{11}(\kappa) \sim H(\kappa)^{-2} \).

One can calculate \( E_{11\rho} \) from built in power-spectral density (PSD) estimation functions in MATLAB, for example, but the researcher must take care when considering the units; factors of 2, \( \pi \), and the period of the signal may appear unintentionally when using built-in PSD functions which will make the amplitude different from Eq. 3.2.1. A standard check is to compute \( \overline{\rho^2} = \int E_{11\rho}(\kappa_1)d\kappa \) for different processing methods to build confidence in the results.

Results for the experiment with a 0.5 arcminute Wollaston prism are presented in Fig. 3.13. The figure shows the response of one of the FLDI beam-pairs in the two-beam pair FLDI setup, corrected by the transfer functions corresponding to the disturbance fields as labeled. The spectra overlap and are therefore offset by a multiple of two from one another along the ordinate for clarity. The model spectrum labeled \( E_{11}(\kappa_1) \) in Fig. 3.13 is the one-dimensional turbulence spectrum given by \( E_{11}(\kappa_1) = \int_{\kappa_1}^{\infty} \frac{E(\kappa)}{\kappa} \left( 1 - \frac{\kappa^2}{\kappa_1^2} \right) \) from Pope [88]. The results indicate that, as the complexity of the modeled disturbance better matches the actual disturbance field, the corrected one-dimensional energy spectra of the density fluctuations more closely match the model spectrum. The transfer functions represent a means to account for the differencing nature of the FLDI (1/\( \kappa \) in Eq. 2.4.10) as well as the response of the FLDI where the disturbance wavenumber, beam size, and overlapping beam area all are on the same order near the focus by introducing spectral components (e.g., \( \sin(\kappa x) \)).
Figure 3.13: One-dimensional energy spectra of the density fluctuations for an experiment performed with a round turbulent jet, corrected by transfer functions for disturbance fields as labeled.

In all cases, the inertial subrange has a slope of $-\frac{5}{3}$ as hypothesized by Kolmogorov. For idealized disturbance fields, where the disturbance in space approaches the limit of infinitely small fluctuations, of the form $\sin(\kappa x)\delta(z)$ or $\sin(\kappa x)\sin(\kappa y)\delta(z)$, the corrections yield an inertial subrange that is shortened by approximately half a decade, resulting in the transition to the dissipation range occurring earlier than expected when compared to the modeled one-dimensional energy spectrum. When the disturbance field more realistically occupies a physical length ($2L$ or $\sigma$), the corrections by the appropriate transfer functions broaden the inertial subrange. The beginning of the dissipation range is not visible for these spectra; a smaller FLDI beam intraspaning would be necessary to process the spectra beyond $\kappa_1\eta \approx 0.1$. 
3.2.3 Conclusion

In this section, application of the derived transfer functions to data collected using FLDI to probe the exit of a round, turbulent jet showed that increasing the complexity of the transfer function has merit. When the disturbance modeled by the transfer function better matches the actual disturbance field, the results obtained from the FLDI instrument align more closely with a well-established model and published data. The best results were obtained when modeling the field to include disturbances in $x$ and $y$ over a physical length scale in $z$, be it $2L$ or $\sigma$. However, modeling the field to include disturbances in $z$ resulted in a transfer function that did not yield meaningful results, most likely due to assumptions about the correlation of disturbances along that integration direction. An alternate treatment where the disturbances are considered in a statistical manner, perhaps a Gaussian random field, could address this issue.
Chapter 4
Measurement of Second-Mode Disturbances in Hypersonic Boundary Layers

Results from a stability investigation using a multi-beam pair FLDI and high-speed schlieren cinematography of hypersonic flow over a cooled and uncooled 5° half-angle cone are presented in this chapter. Experiments were performed at University of Maryland’s HyperTERP reflected shock tunnel using a room-temperature and actively cooled cone. The frequency and phase-speed of the largest-amplitude disturbance (maximum N factor) as predicted by STABL and measured by FLDI or schlieren were in excellent agreement for the room-temperature cases and good agreement for the cooled-wall cases. A comparison between a cooled-wall and room temperature shot at nominally the same Reynolds number shows a later transition to turbulence for the cooled-wall shot.

4.1 Facility and Run Conditions

All experiments were performed in the hypersonic shock tunnel, HyperTERP, operated by the University of Maryland. A schematic of the facility is shown in Fig. 4.1 with major components labeled. The unheated driver section is 3 m long, and the driven section is 10 m long; both have an internal diameter of 100 mm. They are separated by the primary diaphragm station. A double-burst mechanism incorporating two mylar diaphragms is employed to allow accurate control of the burst conditions. The driven section is isolated from the nozzle and downstream components by a secondary mylar diaphragm, just upstream of the nozzle throat. For the experiments performed in this work, a contoured nozzle with an exit diameter of 220 mm and a
design Mach number of 6.0 was manufactured and installed. The nozzle exhausted into a cylindrical test section with an internal diameter of 300 mm.

Figure 4.1: Schematic of the shock tunnel facility employed in the experimental component of this study: (A) driver section; (B) primary (double) diaphragm; (C) driven section; (D) secondary diaphragm; (E) Mach-6 nozzle; (F) test section; (G) dump tank.

The tunnel is typically run under tailored conditions to maximize test time. A typical reservoir pressure trace, showing a constant reservoir pressure for approximately 6 ms is presented in Fig. 4.2. This is shorter than the theoretically predicted test time, a discrepancy that can be attributed to deviations from ideal burst in the double-diaphragm mechanism. The corresponding flow conditions for each of the shots performed in this work are presented in Table 4.1. The reservoir conditions were determined using the procedure described in section 2.6. Error estimates for the test conditions of this facility are provided in Butler and Laurence [89], with shock speed estimates varying ±5 m/s resulting in a 0.4% uncertainty in the reservoir temperature.

The test article for these experiments was a slender, 5° half-angle cone with a nominally sharp nose tip, mounted at zero angle of attack. For the cooled-wall experiments (shots 18, 20-22), the cone was cooled through thermal contact with the immersion probe of the PolyScience IP-100 cooler. This cooler operated by circulating refrigerant within its metallic probe and typically maintained a probe temperature around -95°C. The aluminum frustum of the cone was machined in two halves. The
Figure 4.2: Typical stagnation pressure trace in HyperTERP for this experimental investigation. This is taken from Shot 7. The black line denotes the test time.

Figure 4.3: Predicted surface thermal profile along the actively cooled cone upon reaching thermal equilibrium with the surroundings [90].

cooler probe, 16 mm in diameter, was inserted into the groove through the base of the cone, and thermal paste was applied to increase the amount of thermal contact. A thermocouple measured the surface temperature of the cone 5.2 cm upstream of the base. This system effectively cooled the cone at a rate of about -3°C/min for the first 13 minutes, but the rate of cooling decayed over time due to thermal contact
Table 4.1: Shot conditions. $P_R$, $T_R$, $U_x$, $T_x$, $\rho_x$, $M_x$, $\text{Re}_x^{\text{Unit}}$, $U_E$, $T_E$, $\rho_E$, $M_E$, $\text{Re}_E^{\text{Unit}}$, and $T_W$, are the reservoir pressure, reservoir temperature, exit velocity, exit temperature, exit density, exit Mach number, exit Reynolds number, edge velocity, edge temperature, edge density, edge Mach number, edge unit Reynolds number, and wall temperature, respectively.

<table>
<thead>
<tr>
<th>Shot</th>
<th>$P_R$</th>
<th>$T_R$</th>
<th>$U_x$</th>
<th>$T_x$</th>
<th>$\rho_x$</th>
<th>$M_x$</th>
<th>$\text{Re}_x^{\text{Unit}}$</th>
<th>$U_E$</th>
<th>$T_E$</th>
<th>$\rho_E$</th>
<th>$M_E$</th>
<th>$\text{Re}_E^{\text{Unit}}$</th>
<th>$T_W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.37</td>
<td>785</td>
<td>1187</td>
<td>89</td>
<td>0.026</td>
<td>6.27</td>
<td>5.06e6</td>
<td>1176</td>
<td>102</td>
<td>0.037</td>
<td>5.80</td>
<td>6.06e6</td>
<td>300</td>
</tr>
<tr>
<td>3</td>
<td>1.05</td>
<td>802</td>
<td>1199</td>
<td>91</td>
<td>0.020</td>
<td>6.25</td>
<td>3.75e6</td>
<td>1188</td>
<td>105</td>
<td>0.028</td>
<td>5.78</td>
<td>4.49e6</td>
<td>300</td>
</tr>
<tr>
<td>4</td>
<td>0.76</td>
<td>782</td>
<td>1183</td>
<td>89</td>
<td>0.015</td>
<td>6.23</td>
<td>2.85e6</td>
<td>1172</td>
<td>103</td>
<td>0.021</td>
<td>5.76</td>
<td>3.40e6</td>
<td>300</td>
</tr>
<tr>
<td>5</td>
<td>1.64</td>
<td>795</td>
<td>1196</td>
<td>90</td>
<td>0.031</td>
<td>6.28</td>
<td>5.90e6</td>
<td>1185</td>
<td>103</td>
<td>0.043</td>
<td>5.81</td>
<td>7.07e6</td>
<td>300</td>
</tr>
<tr>
<td>6</td>
<td>2.03</td>
<td>825</td>
<td>1219</td>
<td>93</td>
<td>0.036</td>
<td>6.29</td>
<td>6.85e6</td>
<td>1208</td>
<td>107</td>
<td>0.051</td>
<td>5.82</td>
<td>8.22e6</td>
<td>300</td>
</tr>
<tr>
<td>7</td>
<td>1.32</td>
<td>796</td>
<td>1195</td>
<td>90</td>
<td>0.025</td>
<td>6.26</td>
<td>4.78e6</td>
<td>1184</td>
<td>104</td>
<td>0.035</td>
<td>5.79</td>
<td>5.71e6</td>
<td>300</td>
</tr>
<tr>
<td>10</td>
<td>1.35</td>
<td>798</td>
<td>1197</td>
<td>90</td>
<td>0.025</td>
<td>6.27</td>
<td>4.83e6</td>
<td>1186</td>
<td>104</td>
<td>0.035</td>
<td>5.80</td>
<td>5.79e6</td>
<td>300</td>
</tr>
<tr>
<td>18</td>
<td>1.62</td>
<td>788</td>
<td>1190</td>
<td>89</td>
<td>0.031</td>
<td>6.28</td>
<td>5.91e6</td>
<td>1179</td>
<td>102</td>
<td>0.043</td>
<td>5.81</td>
<td>7.10e6</td>
<td>221</td>
</tr>
<tr>
<td>20</td>
<td>1.93</td>
<td>792</td>
<td>1194</td>
<td>89</td>
<td>0.036</td>
<td>6.29</td>
<td>6.96e6</td>
<td>1183</td>
<td>103</td>
<td>0.050</td>
<td>5.82</td>
<td>8.36e6</td>
<td>226</td>
</tr>
<tr>
<td>21</td>
<td>1.58</td>
<td>888</td>
<td>1265</td>
<td>101</td>
<td>0.026</td>
<td>6.27</td>
<td>4.79e6</td>
<td>1253</td>
<td>116</td>
<td>0.037</td>
<td>5.80</td>
<td>5.77e6</td>
<td>226</td>
</tr>
<tr>
<td>22</td>
<td>1.95</td>
<td>805</td>
<td>1204</td>
<td>91</td>
<td>0.036</td>
<td>6.29</td>
<td>6.88e6</td>
<td>1193</td>
<td>104</td>
<td>0.050</td>
<td>5.82</td>
<td>8.25e6</td>
<td>215</td>
</tr>
</tbody>
</table>

between the cone, sting, and test-section walls. Typically, the temperature of the cone would asymptote around -60°C if the cooler remained operational for 1.5 hours.

The predicted, steady-state thermal gradient across the length of the cone is shown in Fig. 4.3. Thermocouples mounted in the cone’s nose tip and its base indicated agreement within 3.3 K [91].

4.2 Experimental Setup

A multi-beam pair FLDI setup was utilized to characterize the instability within the boundary layer. In this work, the multi-beam pair FLDI setup was generated using additional Wollaston prisms and quarter-wave plates. The components of the FLDI setup are shown in Fig. 4.4.

A single frequency, 532 nm, Cobolt 05-01 laser was used, which was operated at its maximum power output of 1500 mW. An alternating array of three Thorlabs
WPQ20ME-532 quarter-wave plates and three custom-made United Crystals Wollaston prisms stacked in lens tubes attached to a rotation mount was used to orthogonally polarize, split, and orient the beam pairs. A 150 mm focal length converging lens focused the four beam pairs at the center of HyperTERP’s 356 mm wide test section, immediately above the test article.

For shots 2 to 7, the Wollaston prism beam separation angles were $W_1 = 30$ arcminutes, $W_2 = 20$ arcminutes, $W_3 = 2$ arcminutes. For shots 10, 18, and 20, the beam intraspaning was decreased by using a Wollaston prism of a smaller separation angle, $W_3 = 0.5$ arcminutes. Using these components, the beam inter- and intraspaning shown in Fig. 4.5 was achieved for the two Wollaston prism arrangements.

The wall-normal position of the lowest row of beams was carefully adjusted by changing the height of the upbeam converging lens, $C_2$. A measurement of the wall-normal position was gained by raising a razor blade attached to the tip of a dial indicator zeroed at the cone surface. The wall-normal position was recorded once the razor blade passed through the middle of the lowest row of beams. For these experiments, the lower set of beams was located approximately 0.2-0.3 mm above the
4.3 Stability Analysis

Linear-stability diagrams are presented in Figs. 4.6a and 4.6c for shots 10 and 18, respectively, corresponding to a room-temperature (shot 10) and cooled-wall (shot 18) case at the same edge Reynolds number based on the measurement location. The vertical, black line in Figs. 4.6a and 4.6c represents the FLDI or schlieren measurement location along the cone; a slice in frequency-space is taken along this line and the growth rate and phase speed for these respective shots is presented in Figs. 4.6b and 4.6d. Consistent with Mack [13] and many other researchers, it can be seen that wall cooling increases the growth rate and most-amplified frequency, the latter being due to the decreased boundary-layer thickness. A comparison of the STABL calculations...
and the experimental results are summarized in Table 4.2. Instability measurements for Shots 2-7, 10, 18 and 20 were made using the FLDI technique, while schlieren high-speed cinematography was used to perform similar measurements for shots 21 and 22.

Figure 4.6: (a) and (b): Shot 10 - room-temperature cone - $Re_E^{ME} = 2.20e6$, $T_W/T_E = 2.9$. (c) and (d): Shot 18 - cooled-wall cone - $Re_E^{ME} = 2.25e6$, $T_W/T_E = 2.2$. (a) and (c): Linear-stability diagrams. Black line corresponds to measurement location. (b) and (d): growth rate and phase speed at the measurement location denoted by the black line.
4.4 Results and Discussion

In this section, we discuss the measurement of disturbances in a hypersonic boundary layer using schlieren and FLDI for nominally similar cases with and without active wall-cooling. Key parameters from both the measurements and the STABL computations are summarized in Table 4.2. It is observed that STABL does a good job of predicting the frequency of the boundary-layer instability in most cases. In the cases of the cooled-wall, (shots 18, 20-22), STABL slightly under-predicts the frequency content. Further research is required to understand this discrepancy, including a more thorough characterization of the cone surface temperature distribution under cooling. STABL also appears to accurately predict the second-mode phase speed.

Table 4.2: Comparison of FLDI, schlieren, and STABL results. \( M_E \), \( \text{Re}^{\text{Unit}}_E \), \( s_M \), \( \text{Re}^M_E \), State, \( N_{\text{factor}} \), \( \delta \), \( U_E/(2\delta) \), \( f^M \), \( f^S \), \( T_W/T_E \), \( u_{\text{conv}} \), \( c^M = u_{\text{conv}}^M/U_E \), and \( c_{Sr}^S \) are the edge Mach number, the unit Reynolds number, the measurement location, the edge Reynolds number at the point of measurement, the state of the boundary layer (Laminar (L), Instabilities (I), or Turbulent (T)), \( N_{\text{factor}} \) as calculated by STABL at the measurement location, boundary-layer thickness as calculated by DPLR, normalized frequency scale, measured second-mode frequency (B represents broadband turbulence), predicted second-mode frequency by STABL, wall-edge temperature ratio, measured phase speed and normalized measure phase speed, phase speed predicted by STABL, respectively.

<table>
<thead>
<tr>
<th>Shot</th>
<th>( M_E )</th>
<th>( \text{Re}^{\text{Unit}}_E )</th>
<th>( s_M )</th>
<th>( \text{Re}^M_E )</th>
<th>State</th>
<th>( N_{\text{factor}} )</th>
<th>( \delta )</th>
<th>( U_E/(2\delta) )</th>
<th>( f^M )</th>
<th>( f^S )</th>
<th>( T_W/T_E )</th>
<th>( u_{\text{conv}} )</th>
<th>( c^M = u_{\text{conv}}^M/U_E )</th>
<th>( c_{Sr}^S )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5.80</td>
<td>6.06e6</td>
<td>380</td>
<td>2.30e6</td>
<td>T</td>
<td>4.1</td>
<td>2.1</td>
<td>283.6</td>
<td>B</td>
<td>244.1</td>
<td>2.9</td>
<td>1066</td>
<td>0.91</td>
<td>0.92</td>
</tr>
<tr>
<td>3</td>
<td>5.78</td>
<td>4.49e6</td>
<td>380</td>
<td>1.71e6</td>
<td>I</td>
<td>2.9</td>
<td>2.4</td>
<td>250.2</td>
<td>B</td>
<td>240</td>
<td>2.9</td>
<td>1122</td>
<td>0.94</td>
<td>0.95</td>
</tr>
<tr>
<td>4</td>
<td>5.76</td>
<td>3.40e6</td>
<td>380</td>
<td>1.29e6</td>
<td>L</td>
<td>2.6</td>
<td>2.7</td>
<td>215.5</td>
<td>-</td>
<td>182.6</td>
<td>2.9</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>5.81</td>
<td>7.07e6</td>
<td>380</td>
<td>2.69e6</td>
<td>T</td>
<td>4.7</td>
<td>1.9</td>
<td>305.7</td>
<td>B</td>
<td>261.3</td>
<td>2.9</td>
<td>1040</td>
<td>0.88</td>
<td>0.92</td>
</tr>
<tr>
<td>6</td>
<td>5.82</td>
<td>8.22e6</td>
<td>380</td>
<td>3.12e6</td>
<td>T</td>
<td>5.1</td>
<td>1.8</td>
<td>345.1</td>
<td>B</td>
<td>285.0</td>
<td>2.8</td>
<td>1093</td>
<td>0.90</td>
<td>0.91</td>
</tr>
<tr>
<td>7</td>
<td>5.79</td>
<td>5.71e6</td>
<td>380</td>
<td>2.17e6</td>
<td>I</td>
<td>4.0</td>
<td>2.1</td>
<td>280.7</td>
<td>235</td>
<td>237.8</td>
<td>2.9</td>
<td>1120</td>
<td>0.95</td>
<td>0.92</td>
</tr>
<tr>
<td>10</td>
<td>5.80</td>
<td>5.79e6</td>
<td>380</td>
<td>2.20e6</td>
<td>I</td>
<td>4.0</td>
<td>2.1</td>
<td>281.1</td>
<td>245</td>
<td>237.8</td>
<td>2.9</td>
<td>1053</td>
<td>0.89</td>
<td>0.92</td>
</tr>
<tr>
<td>18</td>
<td>5.81</td>
<td>7.10e6</td>
<td>318</td>
<td>2.25e6</td>
<td>I</td>
<td>5.0</td>
<td>1.6</td>
<td>361.8</td>
<td>320</td>
<td>284.9</td>
<td>2.2</td>
<td>965</td>
<td>0.82</td>
<td>0.91</td>
</tr>
<tr>
<td>20</td>
<td>5.82</td>
<td>8.36e6</td>
<td>318</td>
<td>2.65e6</td>
<td>I</td>
<td>5.6</td>
<td>1.5</td>
<td>394.4</td>
<td>350</td>
<td>305.5</td>
<td>2.2</td>
<td>965</td>
<td>0.82</td>
<td>0.91</td>
</tr>
<tr>
<td>21</td>
<td>5.80</td>
<td>5.77e6</td>
<td>318</td>
<td>1.83e6</td>
<td>I</td>
<td>4.3</td>
<td>1.7</td>
<td>359.8</td>
<td>300</td>
<td>284.9</td>
<td>2.0</td>
<td>1114</td>
<td>0.89</td>
<td>0.90</td>
</tr>
<tr>
<td>22</td>
<td>5.82</td>
<td>8.25e6</td>
<td>318</td>
<td>2.62e6</td>
<td>I</td>
<td>5.7</td>
<td>1.5</td>
<td>404.3</td>
<td>355</td>
<td>308.4</td>
<td>2.1</td>
<td>1124</td>
<td>0.94</td>
<td>0.91</td>
</tr>
</tbody>
</table>

In Fig. 4.7, we present two FLDI results: shot 10, where the second-mode insta-
bility was observed throughout the test time, and shot 6, where turbulent broadband response was observed throughout the test time. The spectrograms presented in Fig. 4.7 show evidence of this observation. In the middle chart presented in Fig. 4.7, we present 200 µs of test time where we observe second-mode wave packets and broadband turbulent response as measured by two FLDI detectors separated by a small distance (streamwise beam interspacing given in Fig. 4.5). The bottom chart in Fig. 4.7 shows the correlation of these signals in time, and with accurate measurement of the FLDI beam interspacing, we can get the convective time. Importantly, for beam spacings this small, it is the phase velocity (not group velocity) that is measured by correlation.

In Fig. 4.8, the averaged PSD of the FLDI response for the four FLDI detectors is again presented for two experiments: a laminar case with second-mode disturbances and a turbulent case. The four detectors in Fig. 4.5 nominally show the same response in each case. FLDI detector bundles 1 and 2 and 3 and 4 are located at the same wall-normal location. In Fig. 4.9a, the response of one FLDI probe is presented with increasing Reynolds number. The FLDI probe measures the increasing magnitude of the instability, with eventual broadband turbulent response. Of note is the moderate Reynolds number difference (a factor of 2-3) between incipient instability and transition to turbulence.

To compare the spectral response of the FLDI and schlieren diagnostic techniques, results obtained at nominally the same condition using each technique are presented in Fig. 4.9b. The spectra appear to match very well, bringing confidence in both measurement techniques. The lower noise floor of the FLDI diagnostic allows both first and second harmonics to be discerned, whereas only the first harmonic is weakly visible in the schlieren spectrum.

A comparison between a cooled and room-temperature case is presented in
Fig. 4.10a at nominally the same Reynolds number. The results for the cooled case (shot 18) show appreciably higher frequency content than that for the experiment carried out when the cone was at room temperature (shot 10). An additional cooled/room-temperature comparison is presented in Fig. 4.10b, where two experiments at nominally the same Reynolds number are investigated using the FLDI diagnostic. It is believed that, in general, the room temperature cases would transition to turbulence at a higher Reynolds number than cooled-wall cases because the growth rates of the second-mode instability are typically higher for a cooled wall. This is not borne out in the two experiments shown in Fig. 4.10b. A hypothesis for this counter-intuitive result is: for the cooled-wall cases, even though the growth rates are higher, the most-amplified frequencies which lead to transition are also higher. At higher frequencies, however, the intensity of the free-stream disturbances that ultimately excite the second-mode instability within the boundary layer is lower, potentially leading to delayed transition. This observation is consistent with the N factor calculations in Table 4.2. Moreover, researchers have found that the N factor of transition is correlated to disturbance frequency [92, 93].

4.5 Conclusion

In this chapter, the experimental results obtained using the FLDI and schlieren diagnostic techniques were compared to STABL calculations performed for hypersonic flow over a right-circular cone at zero angle of attack with varying wall-temperature ratio. Specifically, spectral content and phase-speed measurement were discussed for several different cases. Excellent agreement was found between the FLDI and schlieren experimental methods in terms of resolving the spectral content and phase speed of boundary-layer disturbances. For the cooled-wall cases, the observed second-
Figure 4.7: Left: Shot 10 (Re\textsuperscript{M} = 2.20e6) was an experiment where the second mode was observed. Right: Shot 6 (Re\textsuperscript{M} = 3.12e6) shows broadband turbulent response. Top shows spectrograms of run time. Middle shows phase change of FLDI for detectors 1 and 2 separated by a short distance (see Fig 4.5) for short times. Bottom shows the correlation between the FLDI detectors 1 and 2. These lags are used to determine the convective velocity. Red indicates the maximum of a fitted polynomial to the lag data.

Mode boundary-layer instability had higher frequency content. The frequency and phase-speed of the largest-amplitude disturbance (maximum N factor) as predicted by STABL and measured by FLDI or schlieren were also in excellent agreement for the room-temperature cases and good agreement for the cooled-wall cases.

Finally, it was observed that the experiment with the room-temperature wall transitioned to turbulence at the same Reynolds number where the cooled-wall case remained unstable. It is believed that the higher growth rates associated with the cooled-wall cases would result in early transition, relative to the lower growth rates.
Figure 4.8: (a): Shot 3 - $Re^M_E=1.71e6$ shows a case where the second mode is observed, (b): Shot 6 - $Re^M_E=3.12e6$ shows broadband turbulent response. Note that in both Shot 3 and Shot 6 (unstable and turbulent) all four FLDI detector responses are similar.

Figure 4.9: (a): Shots 4, 3, 7, and 6 showing the increased second mode amplitude and broadband response as the Reynolds number at the measurement location ($Re^M_E$) is increased with a fixed wall-to-edge temperature ratio ($T_W/T_E$). (b): At the same Reynolds number and wall-temperature ratio, the schlieren response (Shot 22) and the FLDI response (Shot 20) show excellent agreement in terms of observed second-mode frequency.

for the room-temperature cases. A hypothesis for this counter-intuitive result is that though the growth rates are higher for cooled-wall cases, the frequency that drives the
Figure 4.10: (a): Shots 10 (FLDI) and 18 (FLDI) show that at the same Reynolds number, the observed second-mode frequency is higher for the cooled-wall case (Shot 18) relative to the room-temperature case (Shot 10). (b): For approximately the same Reynolds number, the cool-wall case (Shot 20) shows second-mode boundary-layer instability, while the room-temperature case (Shot 5) shows broadband turbulent response. Boundary layer unstable is also higher. At higher frequencies, there is less wind-tunnel noise, thus the boundary layer transitions at a higher Reynolds number.
Chapter 5
Nonlinear Interactions in a High-Enthalpy Boundary Layer

In this chapter, results are presented for experiments performed at California Institute of Technology’s T5 reflected-shock tunnel. A multi-beam FLDI setup is used to characterize an unstable boundary layer on a 5° cone. Higher-order spectral analysis of the FLDI data is used to determine the degree of coherence between measurements taken inside and outside of the boundary layer. Several nonlinear interactions are revealed through higher-order spectral analysis, including those which contribute to the generation of the first and second harmonics of the second mode within the boundary layer.

5.1 Facility and Experimental Setup

5.1.1 T5 Reflected-Shock Tunnel

The experiments were performed at California Institute of Technology’s T5 free-piston-driven reflected-shock tunnel [94]. In T5, a 120 kg aluminum piston was loaded into the compression tube/secondary reservoir junction. A 127 µm thick Mylar secondary diaphragm was inserted at the nozzle throat, separating the shock tube from the test section prior to the experiment. A 6 mm thick stainless steel primary diaphragm was inserted between the compression tube and the shock tube. Once the facility reached an acceptable level of vacuum, the shock tube was filled with the test gas (ALPHAGAZ air for these experiments), the compression tube was filled with a mixture of 84% Helium and 16% Argon, and the secondary reservoir was pressurized with air. The air in the secondary reservoir was then allowed to push against the
back of the piston, launching it into the compression tube. The accelerating piston
adiabatically compressed the driver gas in the compression tube until the primary
diaphragm ruptured. The rupture of the primary diaphragm caused a shock wave to
propagate into the shock tube, which reflected off the end wall, burst the secondary
diaphragm, and re-processed the test gas to the nozzle reservoir conditions. The test
gas was then expanded through the converging-diverging contoured nozzle to a hyper-
sonic Mach number ($M \approx 5.2$) in the test section. Additional information regarding
the capabilities of T5 can be found in Hornung [94].

The reservoir and freestream run conditions for the shots performed in this
experimental campaign are presented in Table 5.1 and Table 5.2. The freestream
conditions are chosen to be an areal average of the DPLR output at approximately
580 ± 10 mm, the distance from the nozzle’s throat to the location of the cone’s nose
tip. A slightly blunted 5° half-angle cone with two nose tip bluntnesses ($R_N$) was
used as the model in this experimental campaign and the cone was placed at zero
angle of attack in the center of the test section. Error estimates for the calculation
of reservoir and freestream conditions in this facility are provided in Parziale [30].

Table 5.1: Reservoir conditions for Caltech experimental campaign

<table>
<thead>
<tr>
<th>Shot</th>
<th>Gas</th>
<th>$P_R$</th>
<th>$h_R$</th>
<th>$T_R$</th>
<th>$\rho_R$</th>
<th>$y_{N_2}$</th>
<th>$y_{O_2}$</th>
<th>$y_{NO}$</th>
<th>$y_N$</th>
<th>$y_O$</th>
<th>$R_N$</th>
<th>Diag</th>
</tr>
</thead>
<tbody>
<tr>
<td>2983</td>
<td>Air</td>
<td>53.5</td>
<td>9.48</td>
<td>5962</td>
<td>28.3</td>
<td>0.700</td>
<td>0.054</td>
<td>0.135</td>
<td>0.004</td>
<td>0.107</td>
<td>2</td>
<td>Schlieren</td>
</tr>
<tr>
<td>2984</td>
<td>Air</td>
<td>55.8</td>
<td>9.60</td>
<td>6021</td>
<td>29.2</td>
<td>0.699</td>
<td>0.053</td>
<td>0.136</td>
<td>0.005</td>
<td>0.108</td>
<td>2</td>
<td>Schlieren</td>
</tr>
<tr>
<td>2985</td>
<td>Air</td>
<td>58.1</td>
<td>9.03</td>
<td>5794</td>
<td>32.0</td>
<td>0.699</td>
<td>0.064</td>
<td>0.140</td>
<td>0.003</td>
<td>0.095</td>
<td>2</td>
<td>Schlieren</td>
</tr>
<tr>
<td>2986</td>
<td>Air</td>
<td>59.9</td>
<td>8.98</td>
<td>5780</td>
<td>33.1</td>
<td>0.699</td>
<td>0.065</td>
<td>0.140</td>
<td>0.003</td>
<td>0.093</td>
<td>2</td>
<td>Schlieren</td>
</tr>
<tr>
<td>2987</td>
<td>Air</td>
<td>58.9</td>
<td>8.85</td>
<td>5724</td>
<td>33.0</td>
<td>0.699</td>
<td>0.067</td>
<td>0.141</td>
<td>0.003</td>
<td>0.091</td>
<td>2</td>
<td>Schlieren</td>
</tr>
<tr>
<td>2988</td>
<td>Air</td>
<td>60.1</td>
<td>8.93</td>
<td>5758</td>
<td>33.4</td>
<td>0.699</td>
<td>0.066</td>
<td>0.141</td>
<td>0.003</td>
<td>0.092</td>
<td>2</td>
<td>Schlieren</td>
</tr>
<tr>
<td>2989</td>
<td>Air</td>
<td>60.1</td>
<td>9.74</td>
<td>6098</td>
<td>31.0</td>
<td>0.699</td>
<td>0.051</td>
<td>0.136</td>
<td>0.005</td>
<td>0.109</td>
<td>2</td>
<td>Schlieren</td>
</tr>
<tr>
<td>2990</td>
<td>Air</td>
<td>59.6</td>
<td>8.86</td>
<td>5727</td>
<td>33.3</td>
<td>0.699</td>
<td>0.068</td>
<td>0.141</td>
<td>0.003</td>
<td>0.090</td>
<td>2</td>
<td>FLDI</td>
</tr>
<tr>
<td>2991</td>
<td>Air</td>
<td>57.6</td>
<td>8.80</td>
<td>5695</td>
<td>32.4</td>
<td>0.699</td>
<td>0.068</td>
<td>0.140</td>
<td>0.003</td>
<td>0.090</td>
<td>2</td>
<td>FLDI</td>
</tr>
<tr>
<td>2992</td>
<td>Air</td>
<td>59.9</td>
<td>8.44</td>
<td>5552</td>
<td>34.9</td>
<td>0.699</td>
<td>0.077</td>
<td>0.142</td>
<td>0.002</td>
<td>0.081</td>
<td>3</td>
<td>FLDI</td>
</tr>
<tr>
<td>2993</td>
<td>Air</td>
<td>60.7</td>
<td>8.07</td>
<td>5396</td>
<td>36.6</td>
<td>0.700</td>
<td>0.086</td>
<td>0.142</td>
<td>0.001</td>
<td>0.072</td>
<td>3</td>
<td>Schlieren</td>
</tr>
</tbody>
</table>
Table 5.2: Freestream conditions for Caltech experimental campaign

<table>
<thead>
<tr>
<th>Shot</th>
<th>$U_X$ (m/s)</th>
<th>$\rho_X$ (kg/m$^3$)</th>
<th>$P_X$ (kPa)</th>
<th>$T_X$ (K)</th>
<th>$Tv_X$ (K)</th>
<th>$M_X$</th>
<th>$Re_X^*$</th>
<th>$y_{N2}$</th>
<th>$y_{O2}$</th>
<th>$y_{NO}$</th>
<th>$y_N$</th>
<th>$y_O$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2983</td>
<td>3913</td>
<td>0.075</td>
<td>32.1</td>
<td>1495</td>
<td>1480</td>
<td>5.05</td>
<td>5.33e+06</td>
<td>0.733</td>
<td>0.183</td>
<td>0.073</td>
<td>0.000</td>
<td>0.011</td>
</tr>
<tr>
<td>2984</td>
<td>3935</td>
<td>0.078</td>
<td>33.7</td>
<td>1502</td>
<td>5.04</td>
<td>5.48e+06</td>
<td>0.733</td>
<td>0.183</td>
<td>0.073</td>
<td>0.000</td>
<td>0.011</td>
<td></td>
</tr>
<tr>
<td>2985</td>
<td>3839</td>
<td>0.084</td>
<td>33.8</td>
<td>1388</td>
<td>1397</td>
<td>5.11</td>
<td>6.07e+06</td>
<td>0.733</td>
<td>0.186</td>
<td>0.073</td>
<td>0.000</td>
<td>0.008</td>
</tr>
<tr>
<td>2986</td>
<td>3831</td>
<td>0.087</td>
<td>34.6</td>
<td>1379</td>
<td>1387</td>
<td>5.12</td>
<td>6.29e+06</td>
<td>0.733</td>
<td>0.187</td>
<td>0.073</td>
<td>0.000</td>
<td>0.007</td>
</tr>
<tr>
<td>2987</td>
<td>3808</td>
<td>0.086</td>
<td>33.8</td>
<td>1354</td>
<td>1363</td>
<td>5.13</td>
<td>6.28e+06</td>
<td>0.733</td>
<td>0.187</td>
<td>0.073</td>
<td>0.000</td>
<td>0.007</td>
</tr>
<tr>
<td>2988</td>
<td>3822</td>
<td>0.087</td>
<td>34.6</td>
<td>1369</td>
<td>1377</td>
<td>5.12</td>
<td>6.35e+06</td>
<td>0.733</td>
<td>0.187</td>
<td>0.073</td>
<td>0.000</td>
<td>0.007</td>
</tr>
<tr>
<td>2989</td>
<td>3963</td>
<td>0.083</td>
<td>36.6</td>
<td>1523</td>
<td>1530</td>
<td>5.02</td>
<td>5.80e+06</td>
<td>0.733</td>
<td>0.183</td>
<td>0.073</td>
<td>0.000</td>
<td>0.011</td>
</tr>
<tr>
<td>2990</td>
<td>3809</td>
<td>0.087</td>
<td>34.2</td>
<td>1355</td>
<td>1363</td>
<td>5.13</td>
<td>6.35e+06</td>
<td>0.733</td>
<td>0.187</td>
<td>0.073</td>
<td>0.000</td>
<td>0.007</td>
</tr>
<tr>
<td>2991</td>
<td>3797</td>
<td>0.084</td>
<td>32.9</td>
<td>1343</td>
<td>1352</td>
<td>5.14</td>
<td>6.18e+06</td>
<td>0.733</td>
<td>0.187</td>
<td>0.073</td>
<td>0.000</td>
<td>0.007</td>
</tr>
<tr>
<td>2992</td>
<td>3735</td>
<td>0.089</td>
<td>32.5</td>
<td>1265</td>
<td>1274</td>
<td>5.21</td>
<td>6.63e+06</td>
<td>0.732</td>
<td>0.188</td>
<td>0.074</td>
<td>0.000</td>
<td>0.005</td>
</tr>
<tr>
<td>2993</td>
<td>3664</td>
<td>0.092</td>
<td>31.7</td>
<td>1191</td>
<td>1200</td>
<td>5.28</td>
<td>7.04e+06</td>
<td>0.732</td>
<td>0.189</td>
<td>0.076</td>
<td>0.000</td>
<td>0.004</td>
</tr>
</tbody>
</table>

5.1.2 FLDI Setup

The components used to generate the FLDI diagnostic employed in this experimental campaign are shown in Fig. 5.1. The FLDI setup is like that described previously, however, in this campaign, Holo/Or diffractive optical elements were used to interspace the beams in the streamwise and wall-normal directions to provide a greater number of available beam pairs in the wall-normal direction. This resulted in an FLDI diagnostic with a grid of beam pairs two columns wide in the streamwise direction and three rows tall in the wall-normal direction. A picture of the beam pairs taken at their focus using an Ophir-Spiricon LT-665 beam profiler is shown in Fig. 5.2.

The streamwise and wall-normal beam interspacing was 1.7 mm and 1.03 mm, respectively, and the streamwise beam intraspacing was 0.18 mm. The lowest row of beams was positioned at a height of approximately 0.6 mm above the cone’s surface. The streamwise interspaced and intraspaced beams were oriented to be parallel to the cone surface and the wall-normal interspaced beams were oriented to be perpendicular to the cone surface. The boundary-layer thickness and velocity profile were estimated
Figure 5.1: Optical components used to generate the multi-beam pair FLDI diagnostic at the T5 reflected-shock tunnel.

using DPLR. The boundary layer was determined to be approximately 1.1 mm thick at the measurement location of 680 mm along the cone. The velocity profile at this position is represented in Fig. 5.2 as a solid white line. The column of upstream beam pairs and the lowest row of beam pairs were used to interrogate the flow. This selection located two beam pairs within the boundary-layer (FLDI probes C and D) at $y/\delta = 0.6$ and two beam pairs to be at various heights outside of the boundary-layer (FLDI probes B and A at $y/\delta = 1.5$ and $y/\delta = 2.4$, respectively).

5.2 Results and Discussion

Results for shot 2990 are presented in this section. Shot 2990 was an experiment representing a transitional boundary layer with a 2 mm nose-tip at relatively high enthalpy (8.9 MJ/kg) and a wall-to-edge temperature ratio ($T_W/T_E$) of 0.19. Fig. 5.3 shows the reservoir pressure trace for shot 2990, with the red line indicating the average reservoir pressure for the test duration (1.05-2.15 ms). A time delay of approximately 0.5 ms is added to the test duration to account for the flow being accelerated from the reservoir, through the nozzle, and into the test section.
Figure 5.2: Picture of the FLDI beam pairs taken at the focus. The major tick marks are spaced 1 mm apart and the minor tick marks are spaced 0.1 mm apart. The lowest row of beams is located approximately 0.64 mm above the cone surface. The velocity profile is given by the solid white line and the boundary-layer thickness is depicted by the dotted line. The flow is from left to right.

Figure 5.3: Reservoir pressure trace for shot 2990. The red line indicates the average reservoir pressure for the test duration.
5.2.1 Stability Analysis

Results from the stability analysis performed for shot 2990 are presented in this section. The STABL software suite was used to perform the stability analysis; a description of the theory implemented by STABL and the method of extracting relevant data from the computations is provided in section 2.6.

The linear stability diagram for shot 2990 is provided in Fig. 5.4a. The vertical black line in this figure represents the measurement location of the FLDI beams. A slice is taken at this location to obtain Fig. 5.4b, which provides the growth rate and maximum N factor at the measurement location as a function of frequency. From this figure, the most unstable frequency is determined to be 1257 kHz and the most amplified frequency is 1435 kHz. Of particular interest in Fig. 5.4b is the abrupt change in slope that occurs in the growth rate curve at 1459 kHz, leading to a wider range of unstable frequencies. The abrupt change in slope in the growth rate curve was observed by Bitter and Shepherd [19]. They suggest that the lower wall-to-edge temperature ratio allowed the supersonic mode to become and remain unstable, and the discontinuity in the growth rate curve occurred when the dimensionless phase speed \( c_r \) of the fast mode \( F_1 \) became supersonic relative to the freestream \( c_r < 1 - 1/M_e \), where \( M_e \) is the edge Mach number).

Similar to the analysis performed by Bitter and Shepherd [19], the growth rate and dimensionless phase speed for shot 2990 is shown in Fig. 5.5. The black, horizontal dashed lines mark the locations where the dimensionless phase speed equals \( 1 \pm 1/M_e \). A red, vertical dashed line marking the frequency at which the discontinuity occurs in the growth rate curve is extended into the curve of the dimensionless phase speed and intersects it when the phase speed decreases below \( 1 - 1/M_e \) at 1459 kHz. The stability analysis suggests the presence of an unstable supersonic mode for the
Figure 5.4: (a) Linear stability diagram for shot 2990. The vertical black line at 680 mm represents the measurement location of the FLDI beams. (b) Growth rate and maximum N factor extracted at the FLDI measurement location.

Figure 5.5: Spatial growth rate and dimensionless phase speed for shot 2990.
5.2.2 Power Spectral Density

The spectrograms computed from the data collected from the four FLDI probes are plotted in Fig. 5.6. Mack’s second-mode instability is intermittently observed during the test time in the spectrograms for the two FLDI probes within the boundary layer. The second-mode instability is not seen in the spectrograms for the two FLDI probes located outside of the boundary layer. Rather, broadband bursts are observed in these spectrograms at discrete instances in time, with many of these bursts corresponding to instances where the second-mode instability is observed within the boundary layer. Electrical noise is seen in some of these spectrograms, for example in Fig. 5.6d at 1.1 MHz and 2.1 MHz, and is likely from local AM radio stations picked up by the data-acquisition components of the FLDI system.

A short-time Fourier transform is taken from $2335 \mu s \leq t_1 \leq 2380 \mu s$ to further investigate the burst observed during this time by the probes outside of the boundary layer. The resulting short-time PSD is presented in Fig. 5.6e. The FLDI probes within the boundary layer show distinct peaks at $f_0 \approx 1328$ kHz representing Mack’s second-mode instability. Additionally, a peak at $2f_0 \approx 2600$ kHz appears to be the first harmonic of the second-mode instability. Broadband elevated spectral content is also revealed in the short-time PSD for the probes outside of the boundary layer. The black, dashed curve in Fig. 5.6e represents the maximum N factor curve extracted from the stability calculations performed for shot 2990. The most amplified frequency occurs at 1435 kHz, with a maximum N factor of 12.6. This difference in the measured second-mode frequency and the calculated most amplified frequency could be attributed to an error in the calculation of the run condition resulting in an incorrect calculation of the mean flow, or the model having a slight angle of attack relative to the flow.
Figure 5.6: For shot 2990, (a) to (d) spectrograms for FLDI probes A, B, C, and D, respectively. The spectrograms for probes C and D, which are located inside the boundary layer, show the second-mode instability. This feature is not present in the spectrograms for probes A and B. (e) Short-time corrected PSD for all four FLDI probes, showing the second-mode instability at approximately 1328 kHz and its first harmonic measured by probes C and D in the boundary layer.
5.2.3 Higher-Order Spectral Analysis

The features observed in the spectrograms and power spectra computed from data collected from each FLDI probe are further investigated using higher-order spectral analysis to assess the effects of nonlinearity on disturbances inside and outside of the boundary layer. In contrast to the power spectra, higher-order spectra maintain phase information, which allows for the identification of phase-coupled, nonlinear interactions between disturbances. In the present work, the simultaneous collection of data by spatially separated FLDI probes permits the calculation of the cross-bispectrum, which is calculated using the Higher-Order Spectral Analysis (HOSA) toolbox in MATLAB. The cross-bispectrum is defined as,

$$S_{xyz}(f_1, f_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R_{xyz}(\tau, \lambda) e^{-i2\pi f_1 \tau} e^{-i2\pi f_2 \lambda} d\tau d\lambda$$  \hspace{1cm} (5.2.1)$$

where, $R_{xyz}(\tau, \lambda) = E\{x^*(t)y(t+\tau)z(t+\lambda)\}$ is the cross-correlation. Normalizing the cross-bispectrum provides the cross-bicoherence, which is estimated using the `bicoher` function in the HOSA toolbox. There are various methods of normalizing the cross-bispectrum, and the strategy employed in the HOSA toolbox is similar to the one suggested by Brillinger [95]. Therefore, the cross-bicoherence is defined as

$$b_{xyz}(f_1, f_2) = \frac{S(f_1, f_2)}{\sqrt{S(f_1 + f_2)S(f_1)S(f_2)}},$$  \hspace{1cm} (5.2.2)$$

where $S(f)$ is the PSD power. It is estimated by the `bicoher` function as

$$bic(f_1, f_2) = \frac{|B(f_1, f_2)|^2}{P(f_1 + f_2)P(f_1)P(f_2)}.$$  \hspace{1cm} (5.2.3)$$
where $B(f_1, f_2)$ is the averaged estimate of the cross-bispectrum and $P(f)$ are the averaged estimates of the power spectra. The square-root of the output of the \texttt{bicoherx} function is taken to get the estimated cross-bicoherence in the form as defined in Eq. 5.2.2.

Disturbances that are independently excited at the frequency triad, $f_1$, $f_2$, and $f_1 + f_2$ are statistically independent of each other and show no peaks in the cross-bicoherence spectrum. Peaks in the cross-bicoherence spectrum indicate a quadratic phase coupling (QPC) interaction exists between the frequency triad, $f_1$, $f_2$, and $f_1 + f_2$, and the nonlinear interaction $(f_1, f_2) \rightarrow f_1 + f_2$, where the symbol $\rightarrow$ denotes “generated by phase-locked interaction” \cite{35}. The identified phase-locked interactions can either be sum or difference interactions. Sum interactions are associated with the frequency triad $f_1$, $f_2$, and $f_{sum}(= f_1 + f_2)$, and are present in the cross-bicoherence map within the triangle formed by the symmetry line $f_1 = f_2$, the horizontal axis, and $f_n/2$, where $f_n$ is the Nyquist frequency. Difference interactions are associated with the frequency triad $f_1$, $-f_2$, and $f_{difference}(= f_1 - f_2)$, and are present in the region of the cross-bicoherence spectrum bounded by the symmetry line $f_1 = -f_2$, the horizontal axis, and $f_n/2$.

Fig. 5.7 shows the cross-bicoherence maps computed between spatially separated FLDI probes during the burst seen from approximately 2335 $\mu$s to 2380 $\mu$s. Each cross-bicoherence map was generated using the \texttt{bicoherx} function, with a Hanning window applied to 50% overlapping segments 512 points in length. The axes of the cross-bicoherence maps are normalized by the frequency of the second-mode instability, $f_0 \approx 1328$ kHz. The intensity scaling is consistent between all cross-bicoherence maps and limited to show peaks with $b \geq 0.3$ at contour intervals of 0.1. The symmetry lines $f_2 = f_1$ and $f_2 = -f_1$ are outlined on each map. An average of the PSDs of the corresponding FLDI signals is presented to the left and bottom of
each cross-bicoherence map to provide context for the reader.

Fig. 5.7a provides the cross-bicoherence between the two probes within the boundary layer, separated in the streamwise direction by 1.7 mm. This cross-bicoherence map shows the strongest nonlinear interactions. The sum interaction at \((f_0, f_0) \rightarrow 2f_0\) indicates a high level of phase locking between the second mode and high-frequency disturbances, suggesting that the peak observed in the short-time PSDs for the FLDI signals within the boundary layer at \(2f_0 \approx 2600\) kHz is, in fact, the first harmonic of the second mode generated through nonlinear mechanisms [33–36]. The elongated nature of this sum interaction suggests that the sidebands of the second mode are also phase locked, resulting in the spectral broadening that occurs with harmonic generation [35, 37]. This is reflected by the 20% greater full-width half maximum (FWHM) bandwidth of the \(2f_0\) peak compared to the \(f_0\) peak. A weaker sum interaction is observed at \((2f_0, f_0) \rightarrow 3f_0\). Referring to Fig. 5.6e, there is no discernible peak observed in the PSD at \(3f_0\), suggesting that the peak in the cross-bicoherence at this frequency pair represents an early coupling between the second mode and its first harmonic, which contributes to the generation of the second harmonic of the second mode [37].

Weaker sum interactions between the second mode and lower frequencies are also observed for the probes within the boundary layer. Chokani [35] investigated these interactions using digital complex demodulation, which measures the amplitude and phase modulations of a modulated carrier signal as functions of time. The carrier signal, \(x(t)\), is demodulated by the carrier frequency, \(f'_0\), resulting in sum \((f_0 + f'_0)\) and difference \((f_0 - f'_0)\) terms. The difference term is eliminated by setting the demodulation and carrier frequencies to be equal, \(f_0 = f'_0\). A low-pass filter with a pass-band less than the sum frequency term is applied to block the sum frequency component and output the complex demodulate of the carrier signal, from which
Figure 5.7: Cross-bicoherence spectra for shot 2990 during 2335 µs ≤ t₁ ≤ 2380 µs. In these figures, and in subsequent cross-bicoherence maps, the dashed line represents the line of symmetry f₁ = ±f₂. The axis of the cross-bicoherence spectra are normalized by the frequency of the second-mode instability. The pictures at the bottom left of each subfigure show a simplified representation of the FLDI beams. The beam pairs that are being analyzed in each cross-bicoherence spectrum are outlined in the picture.
the amplitude and phase modulation are obtained. Additional mathematical details for digital complex demodulation and its application to interferometry can be found in Choi et al. [96].

The digital complex demodulation technique is applied to the FLDI signals within the boundary layer to ascertain additional information of the nonlinear interaction between the second mode and lower frequencies. The mean-subtracted $\Delta \phi$ response of the FLDI signal is used as the carrier signal and it is demodulated by the previously identified second-mode frequency ($f_0 \approx 1328$ kHz). A zero-phase, low-pass filter with a cutoff frequency of 300 kHz is used to eliminate the sum frequency. The results are presented in Fig. 5.8. The original, carrier signal is shown in the top row and its corresponding amplitude modulate is shown below it. Fig. 5.8a and Fig. 5.8b show results for the upstream and downstream FLDI probes within the boundary layer during the time duration of interest ($2335 \mu s \leq t_1 \leq 2380 \mu s$), when nonlinear interactions between the second mode and lower frequencies were observed. Fig. 5.8c is provided as a reference, and shows the original signal and its amplitude modulate for the upstream FLDI probe within the boundary layer during a time duration when the second-mode instability was not observed to have low-frequency nonlinear interactions. Amplitude modulation is observed in Fig. 5.8a and Fig. 5.8b at $t \approx 2366$ $\mu$s, corresponding to the instance when the amplitude of the second-mode instability is at a local maximum in the spectrogram. In contrast, Fig. 5.8c does not show appreciable amplitude modulation at the time of the second-mode instability’s peak amplitude ($t \approx 2265$ $\mu$s). This suggests that the low-frequency nonlinear interactions that are observed in the cross-bicoherence spectra are associated with amplitude modulation of the second mode [35].

Referring again to Fig. 5.7a, a strong difference interaction is observed at $(2f_0, -f_0) \rightarrow f_0$. This difference interaction provides the nonlinear mechanism through
Figure 5.8: Digital complex demodulation of carrier signals (top) and the resulting amplitude modulates (bottom). Subfigures (a) and (b) are for the upstream and downstream FLDI probes within the boundary layer during the time duration $2335 \mu s \leq t_1 \leq 2380 \mu s$, when nonlinear interactions were observed between the second mode and lower frequency disturbances. Subfigure (c) is for the upstream FLDI probe within the boundary layer during an instance in time when the second mode was observed without any nonlinear interactions.

which energy is exchanged between the second mode and its first harmonic \[35\]. Along with the aforementioned low-frequency interactions, this difference interaction was also observed in the cross-bicoherence spectrum produced from data captured during shot 2988 using high-speed schlieren videography and was identified to be the strongest nonlinear interaction within the boundary layer at this position along the cone \[97\].

The nonlinear interactions observed between the FLDI probes inside and outside of the boundary layer are shown in Fig. 5.7c to Fig. 5.7d. The various cross-bicoherence spectra show peaks occurring at common frequency pairs. There are a few notable interactions that merit further discussion. Namely, in Fig. 5.7c, the range of sum interactions extending from the abscissa to the symmetry line at $0.5f_0$ likely contribute to the elevated low-frequency spectral content observed in the PSD for the FLDI probe at $y/\delta = 2.4$. The difference interaction at $(f_0, -f_0) \to 0$ is observed in many of the cross-bicoherence spectra at various levels of intensity and indicates energy exchange between the second-mode instability and the mean flow \[35\]. Its
intensity is strongest in Fig. 5.7b, which shows the cross-bicoherence map for the upstream FLDI probe within the boundary layer and the FLDI probe positioned immediately above it at $y/\delta = 1.5$. A similar difference interaction is observed in Fig. 5.7c, however, it is weaker due to the increased distance between these FLDI probes. Similarly, the difference interaction between the downstream FLDI probe within the boundary layer and the upstream FLDI probe positioned at $y/\delta = 1.5$ in the upstream column is stronger than its interaction with the FLDI probe positioned at $y/\delta = 2.4$ in the upstream column (Fig. 5.7d and Fig. 5.7e, respectively). However, both interactions are weaker than those observed between the FLDI probes in the upstream column, suggesting a degree of directionality to the interactions.

Nonlinear interactions observed during the timeframe $1805 \mu s \leq t_2 \leq 1850 \mu s$ are presented in Fig. 5.9. As shown in the short-time PSD (Fig. 5.9f), the second-mode instability is weaker in this time duration than previously observed. The weaker second-mode instability results in weaker nonlinear interactions. For the probes within the boundary layer, the sum and difference interactions at $(f_0, f_0) \rightarrow 2f_0$ and $(2f_0, -f_0) \rightarrow f_0$ are also appreciably weaker than those previously observed in Fig. 5.7a. Although the intensity of the sum interaction is weaker, there are still peaks observed at $2f_0$ in the short-time PSDs for the probes within the boundary layer. Additionally, the cross-bicoherence spectrum lacks a peak at $(2f_0, f_0) \rightarrow 3f_0$, suggesting that nonlinear interactions are not involved in the generation of the second harmonic at this time. Despite additional lower frequency difference interactions observed during this timeframe, the sum interaction between the second mode and lower frequency waves is not seen, suggesting the absence of second mode amplitude modulation within this time duration. The cross-bicoherence spectra depicting nonlinear interactions between the FLDI probes inside and outside of the boundary layer (Fig. 5.9c to Fig. 5.9d) show low-frequency nonlinear interactions of similar intensity
to those previously discussed. However, the difference interaction at \((f_0, -f_0) \rightarrow 0\) is replaced with one at \((0.5f_0, -0.5f_0) \rightarrow 0\), suggesting energy exchange between a subharmonic of the second mode and the mean flow.

5.3 Conclusion

In this chapter, FLDI was used to characterize the transitional boundary layer on a 5° half-angle cone. Results from shot 2990 show the second-mode instability measured within the boundary layer at a frequency of approximately 1328 kHz, in good agreement with the most amplified frequency of 1435 kHz obtained using stability analysis and agreeing well with the results obtained using high-speed schlieren in shot 2988, an experiment performed at similar conditions. A short-time PSD centered around a broadband burst seen in the FLDI probes outside of the boundary layer revealed elevated spectral content at low frequencies as well high-frequency peaks. Higher-order spectral analysis was used to determine the degree of coherence between the spatially separated FLDI signals. The cross-bicoherence analysis revealed several sum and difference interactions between the probes within the boundary layer contributing to the generation of the first and second harmonics of the second mode and providing the nonlinear mechanism for energy exchange between the second mode and its first harmonic. These nonlinear interactions identified using FLDI data supported cross-bicoherence results obtained using schlieren data. Weaker sum and difference interactions were observed between FLDI probes inside and outside of the boundary layer. These sum interactions contributed to the elevated low frequency spectral content observed outside of the boundary layer and the difference interactions suggested the exchange of energy between the second mode and the mean flow. Nonlinear interactions observed during another timeframe show similar results, but the weaker
Figure 5.9: Cross-bicoherence spectra for shot 2990 during $1805 \mu s \leq t_2 \leq 1850 \mu s$. The short time PSD of all FLDI probes is shown in (f).
second mode instability during this time duration results in weaker nonlinear inter-
actions.

Despite the variety of experimental conditions tested and flow-diagnostic tech-
niques used, these experiments did not conclusively detect the spontaneous radiation
of sound as was suggested to occur from hypersonic boundary layers subject to high
degrees of wall-cooling [7 17 19 20]. Spectral content was not observed at the
frequency where the supersonic mode was expected to become unstable. Modal re-
construction techniques implemented on high-speed schlieren data collected during
shot 2988 did indicate the radiation of content outside of the boundary layer and
distinct orientation changes within the disturbances, however, the qualitative simi-
larities of these features did not align with the expected quantitative characteristics
of the radiation of the supersonic mode. Further experimental efforts are required to
verify the experimental existence of the supersonic radiation.
Chapter 6

Conclusions and Future Work

Work was performed to further develop the focused laser differential interferometry (FLDI) flow diagnostic technique. A model of FLDI was derived from first principles, considering the local intensity of each beam in an FLDI beam pair. A methodology for developing transfer functions used in the reduction of FLDI data is introduced and is validated by first re-deriving known transfer functions before being used to generate transfer functions for increasingly complex disturbance fields, namely isotropic turbulence. It is shown that careful selection of the integration limits of the idealized FLDI in the denominator of the transfer functions can simplify the FLDI data-reduction procedure by eliminating the need to specify an integration length. Benchtop laboratory experiments performed with a turbulent jet probed by an FLDI indicate that increasing the complexity of the transfer function provides a more accurate representation of the one-dimensional energy spectra of the density fluctuations to the model spectrum by broadening the inertial subrange. Additional laboratory experiments are performed to characterize the response of the FLDI to a controlled phase object, providing a method to calibrate the instrument in both frequency and amplitude.

Experimental campaigns using FLDI as the flow diagnostic were performed at two research facilities. At University of Maryland’s HyperTERP reflected shock tunnel, a multi-beam pair FLDI instrument was developed to measure the phase speed and spectral content of hypersonic boundary-layer instabilities over an uncooled and actively cooled cone. The second-mode instability was observed to have higher spectral content in the actively cooled experiments, supporting the destabilizing influence of the cooled wall. At California Institute of Technology’s T5 reflected shock tun-
nel, extensive experimental campaigns were performed to experimentally verify the
radiation of the supersonic mode in highly cooled boundary layers as predicted by
computational and theoretical results. While the radiation was not observed, nonlin-
ear interactions were found to occur within and extending out of the boundary layer.
Due to the spatial separation of the muti-beam pair FLDI probes, cross-bicoherence
analysis was performed, and indicated the presence of self-sum phase-coupled interac-
tions which led to the generation of the first and second harmonics of the second-mode
instability. Application of digital complex demodulation also revealed the modula-
tion of the second-mode instability by low-frequency phase-coupled waves. Difference
interactions were also identified, and those between FLDI probes inside and outside
of the boundary showed peaks at frequency pairs indicating energy exchange between
the second-mode instability and the mean flow.

As a result of the short streamwise separation distance between the FLDI beam
pairs, the growth of nonlinear interactions was not able to be quantified using cross-
bicoherence analysis. In future studies of nonlinear interactions, it is recommended
the streamwise beam interspacing be increased to provide a greater distance to observe
the growth of nonlinear interactions. The FLDI diagnostic could be placed such that
some beam pairs are upstream of the laminar to turbulent transition location, while
a few are downstream.

Decreasing the intras spacing between FLDI beam pairs allows the instrument
to resolve disturbances of higher wavenumbers. Experiments at Stevens Institute of
Technology are being performed in which the beam interspacing is approximately
30 µm. Combining this short intrasspacing with signal processing techniques to am-
plify the signal above the noise floor allows the FLDI instrument to measure the
dissipation range in a turbulence spectrum. A comparison between experimental and
computational results of the turbulence spectra scaling at various wall-normal heights
within the boundary layer would provide valuable information regarding turbulence modeling. However, such a comparison requires the development of new transfer functions for the FLDI instrument which incorporate solid boundaries above which the density profile changes.
Bibliography


[73] ______, “Static and dynamic characterization of a focused laser differential inter-


Vita

Ahsan Hameed
ahameed@stevens.edu

Objective: To obtain a position in the field of mechanical engineering where my technical experience and research abilities can lead to innovations beneficial to the organization.

Education: Stevens Institute of Technology, Hoboken New Jersey
Master of Engineering in Mechanical Engineering
GPA: 3.87
Stevens Institute of Technology, Hoboken, New Jersey
Bachelor of Engineering in Mechanical Engineering
GPA: 3.90
Honor societies: member of Tau Beta Pi, Pi Tau Sigma
Awards: Edwin A. Stevens Scholarship for academic excellence, American Bureau of Shipping Scholarship for academic excellence, NASA NJSGC Award

Skills: High-performance computing for fluid dynamics (Steven’s OHPC cluster with Eilmer 3, DPLR, STABL), non-intrusive optical diagnostics (differential interferometry) for studying novel flow physics in high-speed boundary layers, reading and interpreting design documents (P&IDs, engineering drawings, datasheets), application of industry codes and standards for equipment specification and design.
Software: STABL (HPC and local), Cantera, DPLR, Eilmer 3, Mathematica, MATLAB, SAP, SolidWorks

Experience: Stevens Institute of Technology, Hoboken, New Jersey
Ph.D. Researcher: June 2018 to Present
• Developed a quad-focused laser differential interferometer (quad-FLDI) setup to measure boundary layer instability.
• Measured the second-mode instability in a hypersonic boundary layer with a cooled and room temperature wall, observing higher frequency content for a cooled wall as well as delayed transition to turbulence due to the higher-most amplified frequencies.
• Aided in the design and manufacturing of components for the Stevens Shock Tunnel, incorporating industry standards such as ASME B31.3
and B16.5 in the design process to develop a safe, robust, and cost-efficient facility.

- Participated in shakedown testing of the Stevens Shock Tunnel to resolve various issues with tunnel operation, including diaphragm material selection for a range of run conditions and run condition measurement.
- Developed a method to formulate transfer functions for the application of the FLDI diagnostic technique to various disturbance profiles. Introduced new, more complex, and more accurate transfer functions for modeling of two-dimensional disturbances with a finite domain. Experimentally tested these transfer functions
- Modeled, designed, and provided technical assurance for the manufacturing of a new nozzle for California Institute of Technology’s T5 Reflected Shock Tunnel. The new nozzle profile eliminated waves that emanated from the existing nozzle’s wall, increasing the facility’s testing capability.

Shell Oil/Motiva Enterprises LLC, Port Arthur, Texas
Rotating Equipment Engineer, FCCU/Alky: December 2012 to May 2016

- Participated in development of a rotating equipment asset management strategy for centrifugal compressors and special purpose steam turbines.
- Developed detailed and fit for purpose rotating equipment scope of work for unit turnaround, utilizing the work performed in the development of a rotating equipment asset management strategy and performance analysis performed on the combustion-air, lift-air, and wet-gas compressor trains.
- Procured funding for project to replace steam separators for special purpose turbines driving critical FCCU rotating equipment to decrease life-cycle maintenance costs.
- Participated in unit turnaround, providing technical assurance for successful repairs to rotating equipment, including the unit’s air blowers, wet-gas compressor, and refrigeration compressor motor.
- Developed and executed plan to address control problems associated with the wet-gas compressor’s anti-surge control valves and controller to increase compressor efficiency.

Shell Oil/Motiva Enterprises LLC, Port Arthur, Texas
Rotating Equipment Engineer, CDU: Feb. 2012 to December 2012
• Procured funding and developed initial design of project to replace lubricating and seal oil skid for vacuum pipe overhead gas compressor.
• Improved reliability of Sier-Bath screw pumps, resulting in an increase of mean time between repairs from 3 months to 1 year.

Shell Oil/Motiva Enterprises LLC, Port Arthur, Texas
Rotating Equipment Engineer, SBU/ASTU: August 2010 to Feb. 2012
• Designed and implemented method to melt and remove hardened sulfur from sulfur pit.
• Established preventative maintenance and repair strategy for ASTU aerators to achieve 90% availability to meet refinery requirements following the refinery’s expansion project.
• Led root-cause failure investigation to identify cause of Garo AB-1500 liquid ring compressors failures, increasing MTBR from 6 months to approximately 2 years.
• Developed method to fabricate Garo AB-1500 liquid ring compressor rotor with aftermarket facility to decrease part cost by 50% and lead time by 33%.

Peer-Reviewed Publications:

Conference Proceedings:


