1. [20]	2. [20]	3. [30]	4. [30]	Total: [100]

Ma 116 **Quiz 2** **October 31, 2006**

Name: *Pipeline Username*:

Check your lecture: ☐ A - P.Dubovski (10:00a) ☐ B - P.Dubovski (11:00p)
☐ C - P.Brady (12:00a)

Check your recitation: ☐ RA - M.Paliwal (Thursday 8:00a) ☐ RD - L.Bussolari (Friday 10:00a)
☐ RB - M.Paliwal (Thursday 9:00a) ☐ RE - L.Bussolari (Friday 11:00a)
☐ RC - M.Paliwal (Thursday 12:00a) ☐ RF - L.Bussolari (Friday 12:00a)

Closed book and closed notes.

Calculators and cellphones are to be stored out of sight during the exam.

Show all of your work. Answers without supporting work may receive no credit.

Pledge and sign: I pledge my honor that I have abided by the Stevens Honor System

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1. [20 pts] Let $\mathbf{a} = \langle 2, -1, 3\rangle$, $\mathbf{b} = \langle -1, 0, 3\rangle$. Find the equation of the plane, which passes through point $A(2, -1, -4)$ and is parallel to both $\mathbf{a}$ and $\mathbf{b}$.
Solution.

$$\mathbf{n} = \mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1 & 3 \\ -1 & 0 & 3 \end{vmatrix} = -3\mathbf{i} - 9\mathbf{j} - \mathbf{k}$$

Then, the equation of the plane is $-3(x-2) - 9(y+1) - (z+4) = 0$, or

$$3x + 9y + z = -7.$$

Answer: $3x + 9y + z = -7$.

2. [20 pts] Find parametric equation of the line through $A(-2,2,4)$ and perpendicular to the plane $2x-y+5z=12$.

Solution.

$\mathbf{n}=\big\langle 2,-1,5\big\rangle$=directional vector of the line.

Answer:

$$\begin{cases} x=-2+2t \\ y=2-t \\ z=4+5t \end{cases}$$

3. [30 pts] Find the length of the curve $\mathbf{r}(t)=\big\langle 2t^{3/2},\cos(4t),\sin(4t)\big\rangle$, $0\le t\le 1$.

Solution.

$$L=\int_0^1 |\mathbf{r}'(t)|dt=\int_0^1 \sqrt{9t+16}dt=\frac{2}{27}(9t+16)^{3/2}\Big|_{t=0}^{t=1}=\frac{2}{27}(125-64)=\frac{122}{27}.$$

4. [30 pts] (a) Find the equation of the tangent line at point $A(1, 1, 2)$ to the curve $\mathbf{r}(t) = t\mathbf{i} + t\mathbf{j} + (1 + t^2)\mathbf{k}$;

Solution.

Points A corresponds to $t = 1$.

$$\mathbf{r}'(t) = \left\langle 1, 1, 2t \right\rangle$$

Hence, $\mathbf{r}'(1) = \left\langle 1, 1, 2 \right\rangle$ is the directional vector of the tangent line passing through $A(1, 1, 2)$. Consequently, the answer is

$$\mathbf{r}(t) = \left\langle 1, 1, 2 \right\rangle + t\left\langle 1, 1, 2 \right\rangle$$

or

$$\begin{cases} x = 1 + t \\ y = 1 + t \\ z = 2 + 2t \end{cases}$$

(b) Find the magnitude of the projection of vector $\mathbf{a} = \left\langle 5, -3, 7 \right\rangle$ onto the above tangent line.

Solution.

$$|\mathrm{proj}_{\mathbf{r}'(1)}\mathbf{a}| = \frac{|\mathbf{a} \cdot \mathbf{r}'(1)|}{|\mathbf{r}'(1)|} = \frac{|5 - 3 + 14|}{\sqrt{6}} = \frac{16}{\sqrt{6}}.$$