1. [20]	2. [20]	3. [20]	4. [20]	5. [20]	Total: [100]

Ma 116 **Quiz 3** **December 4, 2007**

Name: *Pipeline Username:*

Check your lecture: ☐ A - N.Strigul (10:00a) ☐ B - N.Strigul (11:00a)
☐ C - P.Dubovski (12:00p)

Check your recitation: ☐ RA - A.Hofstrom (Thursday 8:00a) ☐ RD - C.McGrath (Friday 10:00a)
☐ RB - A.Hofstrom (Thursday 9:00a) ☐ RE - C.McGrath (Friday 11:00a)
☐ RC - A.Hofstrom(Thursday 12:00p) ☐ RF - C.McGrath (Friday 12:00p)

Closed book and closed notes.

You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.

Pledge and sign: I pledge my honor that I have abided by the Stevens Honor System

1. [20 pts] Given function $f(x,y) = (2x-3y)^3 \ln(1+y^2)$, find
   - gradient $f(x,y)$ at point $A(2,1)$.
   - the directional derivative of $f(x,y)$ at $A(2,1)$ in the direction of point $B(5,-3)$

$$f_x = 3\cdot 2(2x-3y)^2 \ln(1+y^2)$$

$$f_y = 3\cdot(-3)(2x-3y)^2 \ln(1+y^2) + (2x-3y)^3 \frac{2y}{1+y^2}$$

$$f_x(2,1) = 6\ln 2$$

$$f_y(2,1) = -9\ln 2 + 1$$

$$\Rightarrow \nabla f(2,1) = \langle 6\ln 2, 1-9\ln 2\rangle$$

$$\vec{AB} = \langle 3,-4\rangle \Rightarrow \vec{u} = \frac{\vec{AB}}{|\vec{AB}|} = \left\langle \frac{3}{5}, -\frac{4}{5}\right\rangle \text{ - unit vector.}$$

$$D_{\vec{u}} f(2,1) = \nabla f(2,1)\cdot\vec{u} = \frac{18}{5}\ln 2 - \frac{4}{5} + \frac{36}{5}\ln 2 = \frac{54}{5}\ln 2 - \frac{4}{5}.$$

Answer: $\nabla f(2,1) = \langle 6\ln 2, 1-9\ln 2\rangle$; $D_{\vec{u}} f(2,1) = \frac{54}{5}\ln 2 - \frac{4}{5}$

2. [20 pts] (a) Find $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ at point $P(2,1,1)$ if $x^2 y \sin(\frac{\pi x y z}{4}) + yz + z^2 = 6$.

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \quad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} \text{ (implicitely given function } z\text{).}$$

$$F(x,y,z) = x^2 y \sin\left(\frac{\pi x y z}{4}\right) + yz + z^2.$$

$$F_x = 2xy \sin\left(\frac{\pi x y z}{4}\right) + \frac{\pi}{4} x^2 y^2 z \cos\left(\frac{\pi x y z}{4}\right) \Rightarrow F_x(2,1,1) = 4.$$

$$F_y = x^2 \sin\left(\frac{\pi x y z}{4}\right) + \frac{\pi}{4} x^3 z \cos\left(\frac{\pi x y z}{4}\right) + z \Rightarrow F_y(2,1,1) = 5.$$

$$F_z = \frac{\pi}{4} x^3 y^2 \cos\left(\frac{\pi x y z}{4}\right) + y + 2z \Rightarrow F_z(2,1,1) = 3.$$

Then $\frac{\partial z}{\partial x} = -\frac{4}{3}$, $\frac{\partial z}{\partial y} = -\frac{5}{3}$ at $P(2,1,1)$.

(b) Find the equation of tangent plane to the surface $x^2 y \sin(\frac{\pi x y z}{4}) + yz + z^2 = 6$ at point $P(2,1,1)$.

Tangent plane formula: $F_x \cdot (x - x_0) + F_y \cdot (y - y_0) + F_z (z - z_0) = 0$

From the above results we obtain

$$4 \cdot (x-2) + 5(y-1) + 3(z-1) = 0$$

or $4x + 5y + 3z = 16.$

3. [20 pts] Given surface $\vec{r}(u,v)$, what is the normal vector to the tangent plane?

$$\vec{n} = \vec{r}\,'_u \times \vec{r}\,'_v$$

4. [20 pts] Find the critical points for the function $f(x,y) = 3y^3 - 2x^2 - 3y^2 + 4xy$. Which of them are local maxima, minima and saddle points?

$$\begin{cases} f_x = -4x + 4y = 0 \\ f_y = 9y^2 - 6y + 4x = 0 \end{cases} \Rightarrow x = y \Rightarrow 9y^2 - 6y + 4y = 0$$

$$9y^2 - 2y = 0$$

$$y(9y-2) = 0$$

$$y_1 = 0 \quad y_2 = \frac{2}{9}$$

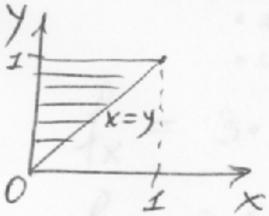
$$x_1 = 0 \quad x_2 = \frac{2}{9}.$$

$$f_{xx} = -4;\ f_{xy} = 4;\ f_{yy} = 18y - 6 \Rightarrow D = f_{xx} \cdot f_{yy} - (f_{xy})^2 = 24 - 72y - 16 = 8 - 72y.$$

At point $A(0,0)$ we obtain $D = 8 > 0$, $f_{xx} = -4 < 0 \Rightarrow$ maximum.

At point $B\left(\frac{2}{9}, \frac{2}{9}\right)$ we obtain $D = -8 < 0 \Rightarrow$ saddle point.

5. [20 pts] Evaluate $\iint\limits_R (3xy)^2 e^{x^3 - y^3}\, dA$ if $R = \{(x,y) :\ 0 \le x \le 1,\ x \le y \le 1\}$.



$$\iint\limits_R = \int_0^1 \int_0^y 9x^2y^2 e^{x^3-y^3}\,dx\,dy = \int_0^1 9y^2 \int_0^y e^{x^3-y^3}\,d(x^3)\cdot\frac{1}{3}$$

$$= \int_0^1 3y^2 \int_0^{y^3} e^{u-y^3}\,du, \text{ where } u = x^3.$$

Then $\displaystyle\iint\limits_R = \int_0^1 3y^2 e^{u-y^3}\Big|_{u=0}^{y^3} dy = \int_0^1 3y^2\left[1 - e^{-y^3}\right]dy = y^3\Big|_{y=0}^{1} - \int_0^1 e^{-y^3} d(y^3)$

$$= 1 + e^{-y^3}\Big|_{y=0}^{1} = e^{-1} = \frac{1}{e}.$$

Answer: $\frac{1}{e}$.