1. [20]	2. [20]	3. [20]	4. [20]	5. [20]	Total: [100]

Ma 116 **Quiz 1** **September 30, 2008**

Name: *Pipeline Username*:

Check your lecture: ☐ A - N.Strigul (10:00a)
☐ B - P.Dubovski (11:00a)
☐ C - P.Dubovski (12:00p)

Closed book and closed notes.
You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.

Pledge and sign: I pledge my honor that I have abided by the Stevens Honor System

1. [20 pts] Does the series $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\,2^n\ln n}{n^3}$ converge? Does the series converge absolutely?

   Solution 1. Using he divergence test, we see $\frac{(-1)^{n-1}2^n\ln n}{n^3}\not\to 0$. Then the series is divergent.
   Solution 2. Using ratio test, we obtain
   $$\left|\frac{a_{n+1}}{a_n}\right| = \frac{2^{n+1}\ln(n+1)\,n^3}{2^n\ln n\cdot(n+1)^3}\left|\frac{a_{n+1}}{a_n}\right| = 2\cdot\frac{\ln(n+1)}{n}\cdot\frac{n^3}{(n+1)^3}\to 2>1.$$
   Since the limit is greater than 1, then the series is divergent. From the usual divergence we instantly obtain the absence of absolute convergence.
   Answer: divergent, not absolutely convergent.

2. [20 pts] Prove that if the series $\displaystyle\sum_{n=1}^{\infty}a_n$ is convergent, then $\lim\limits_{n\to\infty}a_n=0$.

   Solution. Let s_n be partial sum $s_n=\sum\limits_{k=1}^{n}a_k$ and let $\lim\limits_{n\to\infty}s_n=S$. Since $a_n=s_n-s_{n-1}$, then $\lim\limits_{n\to\infty}a_n=\lim\limits_{n\to\infty}(s_n-s_{n-1})=\lim\limits_{n\to\infty}s_n-\lim\limits_{n\to\infty}s_{n-1}=S-S=0$.

3. [20 pts] Does the series $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{n(1+(\ln n)^2)}$ converge? Does the series converge absolutely?

   Solution. First, we check absolute convergence. Since $a_n=\frac{1}{n(1+(\ln n)^2)}\to 0$ monotonically, then we may use the integral test:
   $$\int_1^\infty\frac{dx}{x(1+(\ln x)^2)}=\int_1^\infty\frac{d(\ln x)}{1+(\ln x)^2}=\int_0^\infty\frac{dy}{1+y^2}=\arctan y|_{y=0}^{\infty}=\frac{\pi}{2}.$$
   Consequently, the series is absolutely convergent. Then it is convergent in the usual sense, too.

4. [20 pts] Let $f(x) = \cosh(3x^2)$.
Find the Maclaurin series (i.e., the Taylor series about $x = 0$) for $f(x)$. Find its convergence interval.
Here, by the definition of hyperbolic cosine, $\cosh(z) = \frac{1}{2}\left(e^z + e^{-z}\right)$.
Solution. Since $e^z = \sum\limits_{n=0}^{\infty} \frac{z^n}{n!}$, $z \in (-\infty, +\infty)$, then

$$\cosh z = \frac{1}{2}\sum_{n=0}^{\infty} \frac{z^n}{n!} + \frac{1}{2}\sum_{n=0}^{\infty} \frac{(-1)^n z^n}{n!} = \sum_{k=0}^{\infty} \frac{z^{2k}}{(2k)!}, \quad z \in (-\infty, +\infty).$$

Since the odd terms disappear, we could substitute $n = 2k$ above. We place $3x^2$ for z and obtain the answer:

$$\cosh(3x^2) = \sum_{k=0}^{\infty} \frac{(3x^2)^{2k}}{(2k)!} = \sum_{k=0}^{\infty} \frac{3^{2k} x^{4k}}{(2k)!}, \quad x \in (-\infty, +\infty).$$

5. [20 pts] Find the power series centered at $x_0 = 0$ for function $f(x) = \dfrac{3x}{1 - x^2}$. Find its convergence interval.
Solution.

$$f(x) = 3x\frac{1}{1 - x^2} = 3x\sum_{n=0}^{\infty} x^{2n} = \sum_{n=0}^{\infty} 3x^{2n+1}.$$

The above expression for geometric series works if $x^2 < 1$. Then the convergence interval is $x \in (-1, 1)$.