1. [20]	2. [20]	3. [20]	4. [20]	5. [20]	Total: [100]

Ma 116 **Quiz 2** **October 2008**

Name: *Pipeline Username*:

Check your lecture: □ A - N.Strigul (10:00a)
□ B - P.Dubovski (11:00a)
□ C - P.Dubovski (12:00p)

Closed book and closed notes.
You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.

Pledge and sign: I pledge my honor that I have abided by the Stevens Honor System

1. [20 pts] Find the equation of the plane, which passes through points $A(7,2,-3)$ and $B(5,6,-4)$ parallel to the x-axis.
   Solution. We see that vectors $\vec{i}=\langle 1,0,0\rangle$ and $\vec{AB}=\langle -2,4,-1\rangle$ are parallel to the plane. Then normal vector $\vec{n}=\vec{i}\times\vec{AB}$, and we obtain
   $$\vec{n}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & 0 & 0\\ -2 & 4 & -1\end{vmatrix}=\langle 0,1,4\rangle.$$
   Answer: $y-2+4(z+3)=0$ or $y+4z=-10$.

2. [20 pts] Determine whether the lines are parallel, skew or intersecting. If they intersect, find the points of intersection:
   $$\frac{x-1}{2}=\frac{y-7}{1}=\frac{z-3}{4},\qquad \frac{x-6}{3}=\frac{y+1}{-2}=\frac{z+2}{1}.$$
   Solution. In parametric form we obtain
   $$\begin{cases}x=1+2t\\ y=7+t\\ z=3+4t\end{cases},\qquad \begin{cases}x=6+3s\\ y=-1-2s\\ z=-2+s\end{cases}.$$
   Equalling these equations, we obtain
   $$\begin{cases}1+2t=6+3s\\ 7+t=-1-2s\\ 3+4t=-2+s\end{cases}$$
   Then the solution to the above system $t=-2$, $s=-3$ provides us the intersection point $P(-3,5,-5)$.

3. [20 pts] Find the length of the curve

$$\vec{r}(t) = \langle 8\cos\frac{t}{2}, \frac{15}{7}, 4\sin t\rangle, \quad 0 \le t \le \frac{\pi}{2}.$$

Solution. $L = \int_a^b |\mathbf{r}'(t)|dt$. In our case $\mathbf{r}'(t) = \langle -4\sin\frac{t}{2}, 0, 4\cos t\rangle$. Then

$$L = \int_0^{\pi/2} \sqrt{16\cos^2\frac{t}{2} + 16\cos^2 t}\ dt = 4\int_0^{\pi/2} \sqrt{\cos^2\frac{t}{2} + \cos^2 t}\, dt.$$

4. [20 pts] Find the distance between point $A(1, 2, -3)$ and the plane $4x - 3z - 1 = 0$.
Solution. To find any point from the plane, let $z = 1$ and then $x = 1$. So, point $P(1, 0, 1)$ $\in$plane. The distance is equal to the magnitude of the projection of vector $\vec{AP}$ onto the normal direction to the plane. Since $\vec{AP} = \langle 0, -2, 4\rangle$, $\vec{n} = \langle 4, 0, -3\rangle$,

$$d = \left|\text{comp}_{\vec{n}}\vec{AP}\right| = \left|\frac{\vec{AP}\cdot\vec{n}}{|\vec{n}|}\right| = \frac{12}{5}.$$

5. [20 pts] Let $\mathbf{r}(t)$ be the position vector of a particle. Write formula for its velocity $\mathbf{v}$(t) and acceleration $\mathbf{a}(t)$.

   Solution. $\mathbf{v} = \mathbf{r}'$ and $\mathbf{a} = \mathbf{r}''$.