1. [20]	2. [20]	3. [20]	4. [20]	5. [20]	Total: [100]

Ma 116 **Quiz 3** **December 2, 2008**

Name: *Pipeline Username*:

Check your lecture: ☐ A - N.Strigul (10:00a) ☐ B - P.Dubovski (11:00a)
☐ C - P.Dubovski (12:00p)

Closed book and closed notes.
You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.

Pledge and sign: I pledge my honor that I have abided by the Stevens Honor System

1. [20 pts] Given function $f(x,y) = (x-3y)^3 \ln(1+y)$, find
   - gradient $f(x,y)$ at point $A(2,1)$.
   - the directional derivative of $f(x,y)$ at $A(2,1)$ in the direction of point $B(5,-3)$

   Solution.$\nabla f = \langle f_x, f_y \rangle$.
   $f_x = 3(x-3y)^2 \in (1+y)$; $f_y = -9(x-3y)^2 \in (1+y) + \frac{(x-3y)^3}{1+y}$.
   Then
   $f_x(A) = 3\ln 2,\ f_y(A) = -9\ln 2 - \frac{1}{2}$.

   So, $\nabla f(A) = \langle 3\ln 2, -9\ln 2 - \frac{1}{2} \rangle$.

   $\vec{AB} = \langle 3, -4 \rangle$. Then $\vec{u} = \langle \frac{3}{5}, \frac{-}{4}5 \rangle$. Consequently,

   $D_{\vec{u}} f(A) = \nabla f \cdot \vec{u} = \frac{9}{5}\ln 2 + \frac{36}{5}\ln 2 + \frac{2}{5} = 9\ln 2 + \frac{2}{5}$.

2. [20 pts] (a) Find the equation of tangent plane to the surface $x^2y + yz + z^2 + xz^3 = 8$ at point $P(2,1,1)$.
   Solution.$F(x,y,z) = x^2y + yz + z^2 + xz^3 = 8$.
   $F_x(P) = (2xy + z^3)|_P = 5$; $F_y(P) = (x^2 + z)|_P = 5$, $F_z(P) = (y + 2z + 3xz^2)|_P = 9$.
   Then the tangent plane is $5(x-2) + 5(y-1) + 9(z-1) = 0$ or $5x + 5y + 9z = 24$.

   (b) Find $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$ at point $P(2,1,1)$ if $x^2y + yz + z^2 + xz^3 = 8$.
   Solution.$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{5}{9}$
   $\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{5}{9}$

3. [20 pts] Given surface $\vec{\mathbf{r}}(u,v)$, what is the normal vector to the tangent plane?
   Solution. $\vec{\mathbf{n}} = \vec{\mathbf{r}}_u \times \vec{\mathbf{r}}_v$.

4. [20 pts] Find the critical points for the function $f(x,y) = \frac{2}{3}y^3 + 2x^2y + 4xy$. Which of them are local maxima, minima and saddle points?

Solution.

1. Search for critical points. $f_x = 4xy + 4y = 0$, $f_y = 2y^2 + 2x^2 + 4x = 0$. Then we obtain four critical points $P_1(0,0)$, $P_2(-2,0)$, $P_3(-1,1)$, $P_4(-1,-1)$.

2. Analysis of critical points. $f_{xx} = 4y$, $f_{yy} = 4y$, $f_{xy} = 4x + 4$.
Then $D = 16y^2 - (4x+4)^2$.
$D(P_1) = -16 < 0$. Then P_1 is a saddle point.
$D(P_2) = -16 < 0$. Then P_2 is a saddle point.
$D(P_3) = 16 > 0$, $f_{xx}(P_3) = 4 > 0$. Then P_3 is a minimum point.
$D(P_4) = 16 > 0$, $f_{xx}(P_4) = -4 < 0$. Then P_4 is a maximum point.

5. [20 pts] Evaluate $\iint\limits_R (3xy)^2 dA \qquad$ if $R = \{(x,y) : \ 0 \le x, y \le 1\}$.

Solution.

$$I = \int_0^1 dx \int_0^1 (3xy)^2 dy = 9 \int_0^1 x^2 dx \cdot \int_0^1 y^2 dy = 1.$$