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Formation Control of Nonholonomic Mobile Robots

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Abstract— **In this paper, formation control of a group of nonholonomic wheeled robots are considered. By introducing a unified error of the formation and trajectory tracking, state feedback control laws are proposed for formation control with a desired trajectory. Graph theory and Lyapunov theory are used in the control design. After that, by introducing observers, output feedback control laws are also proposed for the formation control. Simulation study shows the proposed controllers are effective.**

I. Introduction

Cooperative control of multiple systems has received considerable attention recently due to its challenging features and many applications in rescue mission, moving a large object, troop hunting, formation control, and cluster of satellites. Different control strategies have been proposed, which include the behavior based, virtual structure, leader following, and graph theoretical approaches. While most existing results use linear vehicle dynamics to simplify control design, we study formation control of nonholonomic mobile robots, and design nonlinear state and output feedback control for a group of robots to achieve formation on given trajectories.

While a complete review of existing work on cooperative control is beyond of the scope of this paper, we mention a few methods that motivated our research. Arkin studied cooperation without communication for multiple robots foraging and retrieving objects in a hostile environment [1]. In [9], Lewis and Tan proposed the virtual structure concept in formation control of mobile robots. In [3], the authors discussed the problem of coordinating multiple spacecraft to fly in tightly controlled formations using the virtual-structure method. The leader-following approach was used in [4], [11], [18], [19]. Some mobile robots are designated as leaders while others as followers. The leaders track desired trajectories, and the followers track desired trajectories with respect to the leaders. The advantage of this approach is its simplicity in that the reference trajectories of the leaders are pre-defined and the internal stability of the formation are guaranteed by the individual robot's control laws. The graph theory approach was proposed for cooperative control of multiple linear systems by Fax and Murray [5]. Then, different control laws were designed with the aid of graph theory [8], [13]. Communication links among systems are described by Laplacian matrices. Each vehicle is treated as a vertex and the communication links between vehicles are treated as edges. Stability of the whole system is guaranteed by stability of each modified individual linear system. However, the methods are limited to linear systems.

The consensus problem is closely related to cooperative control and has been widely discussed recently. In [6], cooperative laws were proposed using nearest neighbor rules. In [10], it was shown how to make a group of mobile robots converge to a line or general geometric form by solving the consensus problem. In [14], the consensus problem for networks of dynamic agents with fixed and switching topologies was discussed. Two consensus protocols for networks with and without time-delays were proposed for convergence analysis in different communication cases. In [15], [17], the authors considered the problem of information consensus among multiple agents in the presence of limited and unreliable information exchange with dynamically changing interaction topologies. Updated algorithms were proposed for information consensus in both discrete and continuous cases. In addition, in [20], a distributed smooth time-varying feedback control law is proposed for coordinating motions of multiple nonholonomic mobile robots of the Hilare-type to capture/enclose a target by making troop formations with the aid of averaging theory.

In this paper, we discuss the cooperative control of a group of mobile robots with a given formation and a desired trajectory as a group. In order to solve the formation control problem, we first introduce a unified error which consists of the formation error and the tracking error. It is shown that the mobile robots come into formation and move along the desired trajectory if the unified error converges to zero. Based on the dynamics of the unified error, a state feedback control law is proposed for each robot, which renders the team into formation and asymptotically moves the team along the desired trajectory. After that, we extend the design to the output feedback for situations where full states are not available for control. By introducing an observer for each robot, we design an output feedback controller for each robot to achieve the same control objective. In contrast to existing results on linear vehicle systems, we focus on nonholonomic vehicle dynamics and design nonlinear control laws to achieve formation using Lyapunov techniques. Simulations show satisfactory performances.

II. Problem Statement

Consider m mobile robots which are moving on a plane. Without lose of generality, the mobile robots are indexed with $1,\ 2,\ \ldots,\ m$. For simplicity, we assume that each member of the group of mobile robots has the same mechanical structure, i.e., they have the same kinematic model except

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that the geometric parameters may be different. By the existing results on the mobile robots in [12], the kinematics of the group of mobile robots are equivalent to the following canonical form by global or local state diffeomorphisms and input transformations

$$\left\{\begin{array}{lll} \dot{q}_{1j} & = & u_{1j} \\ \dot{q}_{ij} & = & u_{1j}q_{i+1,j}, 2 \leq i \leq n-1, 1 \leq j \leq m \\ \dot{q}_{nj} & = & u_{2j} \end{array}\right. \tag{1}$$

where q_{ij} $(1 \leq i \leq n)$ are the states of robot j and u_{1j} and u_{2j} are the control inputs of the robot j. Specifically, (q_{1j}, q_{2j}) are the position (x_j, y_j) of robot j in the fixed world coordinate system. States q_{ij} $(3 \leq i \leq n)$ are the remaining states of robot j.

Represent the m mobile robots as m vertices in $\mathcal{V}$ of a graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$. The communication between the robots can be described by the edge $\mathcal{E}$ of the graph $\mathcal{G}$. An edge $(i, j) \in \mathcal{E}$ means that the state of robot i is available to robot j. Let $\mathcal{N}_j$ be a collection of neighbors of robot j, i.e., a set of indexes of robots whose states are available to robot j. The information available for robot j in the control are only states of robot j and robot i for $i \in \mathcal{N}_j$. Due to sensor range limitations and bounded bandwidth of communications between robots, the topology of the graph G may vary from time to time, which means $\mathcal{N}_j$ is time-varying.

In the plane, the geometric form of the formation can be described either by relative positions between the robots or by position vectors of robots [10]. In the later approach, a desired formation $\mathcal{F}$ is described by the vector offset denoted by (h_{jx}, h_{jy}) for vehicle j. The offset after rotation and translation is relative to a time varying centroid trajectory $\mathcal{T}$ where the centroid trajectory $\mathcal{T}$ is generated by the virtual mobile robot

$$\left\{\begin{array}{lll} \dot{q}_{1d} & = & u_{1d} \\ \dot{q}_{id} & = & u_{1d}q_{i+1,d}, 2 \leq i \leq n-1 \\ \dot{q}_{nd} & = & u_{2d} \end{array}\right. \tag{2}$$

where q_{id} $(1 \leq i \leq n)$, u_{1d}, and u_{2d} are known time-varying functions. Our control problem is defined as follows.

Formation Control Problem: Design a controller for each robot based on its and its neighbor's states such that the group of robots comes into formation $\mathcal{F}$ and the center of the group of robots moves along the desired trajectory $\mathcal{T}$. That is, design control laws (u_{1j}, u_{2j}) which are functions of q_{ij} and q_{ik} for $k \in \mathcal{N}_j$ and $1 \leq i \leq n$ such that the group of robots converges to the desired geometric formation $\mathcal{F}$, i.e.,

$$\lim_{t\to\infty}(q_{1j}(t) - q_{1i}(t)) = h_{jx} - h_{ix}, (1 \leq i \neq j \leq m) \tag{3}$$
$$\lim_{t\to\infty}(q_{2j}(t) - q_{2i}(t)) = h_{jy} - h_{iy}, (1 \leq i \neq j \leq m), \tag{4}$$

and the formation centroid moves along the desired trajectory $\mathcal{T}$, i.e.,

$$\lim_{t\to\infty}\left(\frac{1}{m}\sum_{j=1}^{m} q_{1j}(t) - q_{1d}(t)\right) = 0 \tag{5}$$
$$\lim_{t\to\infty}\left(\frac{1}{m}\sum_{j=1}^{m} q_{2j}(t) - q_{2d}(t)\right) = 0. \tag{6}$$

In order to solve the formation control problem, we make the following assumption on u_{1d}.

Assumption 1: $\lim_{t\to\infty} \inf |u_{1d}(t)| = \epsilon > 0$.
This assumption is not unusual and easily satisfied in practical control.

In (1), we assume without loss of generality that q_{1j} and q_{2j} are the X and Y coordinates of robot j. This assumption is purely for the convenience of presentation. Without this assumption, similar results can still be developed.

III. Formation Controller Design

In this section, we design a formation controller for each robot based on its own state and that of its neighbors. To this end, define the variable transformation

$$\left\{\begin{array}{l} z_{1j} = q_{1j} - q_{1d} - h_{jx} + \frac{1}{m}\sum_{l=1}^{m} h_{lx} \\ z_{2j} = q_{2j} - q_{2d} - h_{jy} + \frac{1}{m}\sum_{l=1}^{m} h_{ly} \\ z_{ij} = q_{ij} - q_{id} + \alpha_{ij} (3 \leq i \leq n, 1 \leq j \leq m) \end{array}\right. \tag{7}$$

where

$$\left\{\begin{array}{l} \alpha_{3j} = k_2 u_{1d}^{2\rho-1} z_{2j} \\ \alpha_{ij} = k_{i-1} u_{1d}^{2\rho-1} z_{i-1,j} - z_{i-2,j} + z_{i+1,j} \\ +\displaystyle\sum_{l=0}^{i-4} \frac{\partial \alpha_{i-1,j}}{\partial u_{1d}^{[l]}} \frac{u_{1d}^{[l+1]}}{u_{1d}} + \frac{\partial \alpha_{i-1,j}}{\partial z_{2j}}(-k_2 u_{1d}^{2\rho-1} z_{2j} \\ +z_{3j}) + \displaystyle\sum_{l=3}^{i-2} \frac{\partial \alpha_{i-1,j}}{\partial z_{lj}}(-k_l u_{1d}^{2\rho-1} z_{lj} \\ -z_{l-1,j} + z_{l+1,j}), (4 \leq i \leq n) \end{array}\right. \tag{8}$$

where k_i $(2 \leq i \leq n-1)$ are positive constants and $\rho = n-2$, we have

$$\left\{\begin{array}{l} \dot{z}_{1j} = u_{1j} - u_{1d} \\ \dot{z}_{2j} = -k_2 u_{1d}^{2\rho} z_{2j} + u_{1d} z_{3j} + (u_{1j} - u_{1d})\beta_{2j} \\ \dot{z}_{3j} = -k_3 u_{1d}^{2\rho} z_{3j} + u_{1d}(z_{4j} - z_{2j}) + (u_{1j} - u_{1d})\beta_{3j} \\ \quad\vdots \qquad\qquad\qquad (1 \leq j \leq m) \\ \dot{z}_{n-1,j} = -k_{n-1} u_{1d}^{2\rho} z_{n-1,j} - u_{1d} z_{n-2,j} + u_{1d} z_{nj} \\ \qquad + (u_{1j} - u_{1d})\beta_{n-1,j} \\ \dot{z}_{nj} = u_{2j} - u_{2d} + \dfrac{\partial \alpha_{nj}}{\partial z_{2j}}(-k_2 u_{1d}^{2\rho-1} z_{2j} + z_{3j}) \\ \qquad + \displaystyle\sum_{l=3}^{n-2} \frac{\partial \alpha_{nj}}{\partial z_{lj}}(-k_l u_{1d}^{2\rho-1} z_{lj} - z_{l-1,j} + z_{l+1,j}) \\ \qquad + (u_{1j} - u_{1d})\beta_{nj} \end{array}\right. \tag{9}$$

where

$\beta_{2j} = z_{3j} - \alpha_{3j} + q_{3d}$,
$\beta_{ij} = z_{i+1} - \alpha_{i+1} + q_{i+1,d} + \sum_{l=2}^{i-1} \frac{\partial \alpha_{ij}}{\partial z_{lj}} \beta_{lj}$, $(3 \leq i \leq n)$.

Lemma 1: For the state variable transformation (7), if $\lim_{t\to\infty} z_{1j} = 0$ and $\lim_{t\to\infty} z_{2j} = 0$ for $1 \le j \le m$, then (3)-(6) are satisfied.

Proof: Simple calculation can derive the results. ■

Variables z_{1j} and z_{2j} $(1 \le j \le m)$ are unified errors of formation and trajectory tracking between each robot and the desired trajectory. By Lemma 1, the formation control problem can be solved by designing a control law for system (9) such that z_{1j} and z_{2j} converge to zero, respectively.

By the structure of (9), the controller can be designed in two steps. In the first step, we design u_{1j} such that z_{1j} $(1 \le j \le m)$ converge to zero. In the second step, we design u_{2j} such that z_{2j} $(1 \le j \le m)$ converge to zero. Based on the first equation of (9), we have the following lemma.

Lemma 2: For system (9), control laws

$$u_{1j} = -k_1 z_{1j} - \sum_{i\in\mathcal{N}_j} a_{ji}(z_{1j} - z_{1i}) + u_{1d}, (1 \le j \le m) \quad (10)$$

make $\lim_{t\to\infty} z_{1j} = 0$ where constants $k_1 > 0$ and $a_{ij} > 0$ for $1 \le j \le m$.

Proof: The closed-loop z_1 dynamic system can be written as

$$\dot{z}_{1.} = -k_1 z_{1.} - L_{\mathcal{G}} z_{1.} \quad (11)$$

where $L_{\mathcal{G}}$ is the weighted Laplacian matrix [5], [14], $z_{1.} = [z_{11}, \ldots, z_{1m}]$.

Since the sum of each row of $L_{\mathcal{G}}$ is zero, $-k_1 I - L_{\mathcal{G}}$ is a diagonal dominant matrix with negative elements in the diagonal. So, the eigenvalues of $-k_1 I - L_{\mathcal{G}}$ are negative. Therefore, (11) is asymptotically stable, i.e., $\lim_{t\to\infty} z_{1j} = 0$ for $1 \le j \le m$. ■

Next, we design control law u_{2j}. With the aid of the structure of (9), we have the following lemma.

Lemma 3: For system (9), under Assumption 1, if u_{1j} $(1 \le j \le m)$ are chosen as (10), control laws

$$\begin{aligned} u_{2j} = &-k_n z_{nj} - \sum_{i\in\mathcal{N}_j} a_{ji}(z_{nj} - z_{ni}) - \frac{\partial \alpha_{nj}}{\partial z_{2j}}(z_{3j} \\ &-k_2 u_{1d}^{2\rho-1} z_{2j}) - \sum_{l=3}^{n-2} \frac{\partial \alpha_{nj}}{\partial z_{lj}}(z_{l+1,j} - z_{l-1,j} \\ &-k_l u_{1d}^{2\rho-1} z_{lj}) + u_{2d} - (u_{1j} - u_{1d})\beta_{nj} \end{aligned} \quad (12)$$

$(1 \le j \le m)$ make $\lim_{t\to\infty} z_{kj} = 0$, $(2 \le k \le n, 1 \le j \le m)$ where constant $k_n > 0$.

Proof: With control law (12), we have

$$\dot{z}_{nj} = -k_n z_{nj} - \sum_{i\in\mathcal{N}_j} a_{ji}(z_{nj} - z_{ni}), 1 \le j \le m \quad (13)$$

By the same method as for Lemma 2, we can prove that $\lim_{t\to\infty} z_{nj}(t) = 0$.

Next, we prove that $\lim_{t\to\infty} z_{ij}(t) = 0$ for $2 \le i \le n-1$. Let the positive definite function $V = \frac{1}{2}\sum_{i=2}^{n-1} z_{ij}^2$. Differentiating V along (9), we have

$$\dot{V} = -\sum_{i=2}^{n-1} k_i u_{1d}^{2\rho} z_{ij}^2 + \sum_{i=2}^{n-1} (u_{1j} - u_{1d}) z_{ij}\beta_{ij} + u_{1d} z_{n-1,j} z_{nj}.$$

With the control laws (10) and (12), $(u_{1j} - u_{1d})$ and z_{nj} converge to zero by Lemmas 1 and 2. Noting the expressions of β_{ij}, $\dot{V} \le -2k_{min} u_{1d}^{2\rho} V + \xi_1(t)V + \xi_2(t)\sqrt{V}$ where $k_{min} = \min\{k_i, 2 \le i \le n-1\}$, $\xi_1(t)$ and $\xi_2(t)$ are non-negative and converge to zero. Let $\chi = \sqrt{V}$, we have $\dot{\chi} \le (-k_{min} u_{1d}^{2\rho} + \xi_1(t)/2)\chi + \xi_2(t)/2$. Noting Assumption 1, χ tends to zero which implies that V and z_{ij} $(2 \le i \le n-1)$ tend to zero. Therefore, $\lim_{t\to\infty} z_{ij} = 0$ for $2 \le i \le n$. ■

By Lemmas 2-3, we have the following theorem.

Theorem 1: For system (1), under Assumption 1, control laws (10) and (12) make (3)-(6) satisfied, where the control parameters are chosen as in Lemmas 2-3.

Proof: With control laws (10) and (12), z_{1j} and z_{2j} converge to zero. By Lemma 1, (3)-(6) are satisfied. ■

In Theorem 1, the formation control problem is solved. Because the z_{ij} unified error variables combine trajectory and formation relative information, by driving the z_{ij} variables to zero the control law simultaneously maintains the formation and drives the formation along the trajectory using self-state and local communication only between neighbors. In control laws (10) and (12), the control parameters are k_i (> 0) and a_{ij} (> 0). The convergence rate of the formation errors is dependent on k_i $(1 \le i \le n)$, the a_{ij}'s, and the communication topology among the group of robots. Generally, larger values of k_i make the tracking and formation errors converge to zero faster. Delays in the communication affect the stability of the closed-loop system. However, that analysis is beyond the space limitations of this paper.

IV. Output Feedback Formation Control

In this section, we discuss the formation control using output feedback. We assume the measured outputs are $(q_{1,j}, q_{2,j})$ for each robot j and the information used in the control are only $(q_{1,j}, q_{2,j})$ for $1 \le j \le m$. To this end, we first design an observer. Let

$$\xi_{ij} = q_{i+2,j} - L_{ij} q_{2j}, 1 \le i \le n-2, 1 \le j \le m \quad (14)$$

where L_{ij} are constant parameters to be designed later. For each j, we have

$$\begin{cases} \dot{\xi}_{1j} = (\xi_{2j} + L_{2j} q_{2j}) u_{1j} - L_{1j}(\xi_{1j} + L_{1j} q_{2j}) u_{1j} \\ \quad \vdots \qquad 1 \le j \le m \\ \dot{\xi}_{n-3,j} = (\xi_{n-2,j} + L_{n-2,j} q_{2j}) u_{1j} - L_{n-3,j}(\xi_{1j} \\ \qquad + L_{1j} q_{2j}) u_{1j} \\ \dot{\xi}_{n-2,j} = u_{2j} - L_{n-2,j}(\xi_{1j} + L_{1j} q_{2j}) u_{1j} \end{cases} \quad (15)$$

Motivated by the Luenberger observer design, we introduce the following observer for (15) ([7])

$$\begin{cases} \dot{\widehat{\xi}}_{1j} = (\widehat{\xi}_{2j} + L_{2j} q_{2j}) u_{1j} - L_{1j}(\widehat{\xi}_{1j} + L_{1j} q_{2j}) u_{1j} \\ \quad \vdots \qquad 1 \le j \le m \\ \dot{\widehat{\xi}}_{n-3,j} = (\widehat{\xi}_{n-2,j} + L_{n-2,j} q_{2j}) u_{1j} - L_{n-3,j}(\widehat{\xi}_{1j} \\ \qquad + L_{1j} q_{2j}) u_{1j} \\ \dot{\widehat{\xi}}_{n-2,j} = u_{2j} - L_{n-2,j}(\widehat{\xi}_{1j} + L_{1j} q_{2j}) u_{1j} \end{cases} \quad (16)$$

Let $\tilde{\xi}_j = [\tilde{\xi}_{1j}, \dots, \tilde{\xi}_{n-2,j}] = [\xi_{1j} - \widehat{\xi}_{1j}, \dots, \xi_{n-2,j} - \widehat{\xi}_{n-2,j}]$ for $1 \le j \le m$, we have

$$\dot{\tilde{\xi}}_j = u_{1j} A_j \tilde{\xi}_j, 1 \le j \le m \tag{17}$$

where

$$A_j = \begin{bmatrix} -L_{1j} & 1 & 0 & \cdots & 0 \\ -L_{2j} & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -L_{n-3,j} & 0 & 0 & \cdots & 1 \\ -L_{n-2,j} & 0 & 0 & \cdots & 0 \end{bmatrix}$$

Lemma 4 ([7]): If $u_{1j} - u_{1d}(1 \le j \le m)$ converges to zero and $u_{1d}(\ge 0)$ satisfies Assumption 1, then $\tilde{\xi}_j(1 \le j \le m)$ converge to zero, where L_{ij} are chosen such that A_j is asymptotically stable.

By Lemma 4, (16) is an observer of (15) if u_{1j} satisfy assumptions in Lemma 4. Noting (14), we can easily obtain the estimate of q_{ij} $(3 \le i \le n, 1 \le j \le m)$.

Introduce the following variables

$$\begin{cases} e_{1j} = q_{1j} - q_{1d} - h_{jx} + \frac{1}{m}\sum_{l=1}^{m} h_{lx} \\ e_{2j} = q_{2j} - q_{2d} - h_{jy} + \frac{1}{m}\sum_{l=1}^{m} h_{ly} \\ e_{3j} = \widehat{\xi}_{1j} - (q_{3d} - L_{1j}(q_{2d} + h_{jy} - \frac{1}{m}\sum_{l=1}^{m} h_{ly})) \\ \quad\vdots \qquad 1 \le j \le m \\ e_{nj} = \widehat{\xi}_{n-2,j} - (q_{nd} - L_{n-2,j}(q_{2d} + h_{jy} - \frac{1}{m}\sum_{l=1}^{m} h_{ly})) \end{cases} \tag{18}$$

we have

$$\begin{cases} \dot{e}_{1j} = u_{1j} - u_{1d} \\ \dot{e}_{2j} = (e_{3j} + L_{1j}e_{2j})u_{1d} + \tilde{\xi}_{1j}u_{1j} + (e_{3j} + q_{3d} \\ \qquad + L_{1j}e_{2j})(u_{1j} - u_{1d}) \\ \dot{e}_{3j} = (e_{4j} + L_{2j}e_{2j} - L_{1j}(e_{3j} + L_{1j}e_{2j}))u_{1d} \\ \qquad + (e_{4j} + L_{2j}e_{2j} - L_{1j}(e_{3j} + L_{1j}e_{2j}) \\ \qquad + q_{4d} - L_{1j}q_{3d})(u_{1j} - u_{1d}) \\ \quad\vdots \qquad 1 \le j \le m \\ \dot{e}_{n-1,j} = (e_{nj} + L_{n-2,j}e_{2j} - L_{n-3,j}(e_{3j} + L_{1j}e_{2j}))u_{1d} \\ \quad + (e_{nj} + L_{n-2,j}e_{2j} - L_{n-3,j}(e_{3j} + L_{1j}e_{2j}) + q_{nd} \\ \quad - L_{n-3,j}q_{3d})(u_{1j} - u_{1d}) \\ \dot{e}_{nj} = u_{2j} - L_{n-2,j}(\widehat{\xi}_{1j} + L_{1j}q_{2j})u_{1j} - u_{2d} \\ \qquad + L_{n-2,j}q_{3d}u_{1d} \end{cases} \tag{19}$$

By the transformation

$$\begin{cases} z_{1j} = e_{1j}, z_{2j} = e_{2j} \\ z_{ij} = e_{ij} - \alpha_{ij}, 3 \le i \le n, 1 \le j \le m \end{cases} \tag{20}$$

where

$$\begin{cases} \alpha_{3j} = -k_2 u_{1d}^{2\rho-1} z_{2j} - L_{1j}z_{2j} \\ \alpha_{ij} = -k_{i-1}u_{1d}^{2\rho-1} z_{i-1,j} - z_{i-1,j} - L_{i-2,j}e_{2j} \\ +L_{i-3,j}(e_{3j} + L_{1j}e_{2j}) + \dfrac{\partial \alpha_{i-1,j}}{\partial e_{2j}}(e_{3j} + L_{1j}e_{2j}) \\ +\sum_{l=3}^{i-2} \frac{\partial \alpha_{i-1,j}}{\partial e_{lj}}(e_{l+1,j} + L_{l-1,j}e_{2j} - L_{l-2,j}(e_{3j} \\ +L_{1j}e_{2j})) + \sum_{l=0}^{i-4} \frac{\partial \alpha_{i-1,j}}{\partial u_{1d}^{[l]}} \frac{u_{1d}^{[l+1]}}{u_{1d}}, (4 \le i \le n) \end{cases} \tag{21}$$

k_i $(2 \le i \le n-1)$ are constants and $\rho \ge n-2$, we have

$$\begin{cases} \dot{z}_{1j} = u_{1j} - u_{1d} \\ \dot{z}_{2j} = -k_2 u_{1d}^{2\rho} z_{2j} + u_{1d}z_{3j} + u_{1j}\tilde{\xi}_{1j} + \beta_{1j}(u_{1j} - u_{1d}) \\ \dot{z}_{3j} = -k_3 u_{1d}^{2\rho} z_{3j} - u_{1d}z_{2j} + u_{1d}z_{4j} - \dfrac{\partial \alpha_{3j}}{\partial z_{2j}} u_{1d}\tilde{\xi}_{1j} \\ \qquad + \beta_{2j}(u_{1j} - u_{1d}) \\ \quad\vdots \qquad 1 \le j \le m \\ \dot{z}_{n-1,j} = -k_{n-1}u_{1d}^{2\rho} z_{n-1,j} - u_{1d}z_{n-2,j} + u_{1d}z_{nj} \\ \qquad - \dfrac{\partial \alpha_{n-1,j}}{\partial z_{2j}} u_{1d}\tilde{\xi}_{1j} + \beta_{n-2,j}(u_{1j} - u_{1d}) \\ \dot{z}_{nj} = u_{2j} - L_{n-2,j}(\widehat{\xi}_{1j} + L_{1j}q_{2j})u_{1j} - u_{2d} \\ \qquad + L_{n-2,j}q_{3d}u_{1d} - \displaystyle\sum_{l=0}^{n-1} \frac{\partial \alpha_{nj}}{\partial u_{1d}^{[l]}} u_{1d}^{[l+1]} \\ \qquad - \dfrac{\partial \alpha_{nj}}{\partial e_{2j}}((e_{3j} + L_{1j}e_{2j})u_{1d} + \tilde{\xi}_{1j}u_{1j}) \\ \qquad - \displaystyle\sum_{l=3}^{n-1} \frac{\partial \alpha_{nj}}{\partial e_{lj}}(e_{l+1,j} + L_{l-1,j}e_{2j} \\ \qquad - L_{l-2,j}(e_{3j} + L_{1j}e_{2j}))u_{1d} + \beta_{n-1,j} \end{cases} \tag{22}$$

where

$$\begin{cases} \beta_{1j} = e_{3j} + q_{3d} + L_{1j}e_{2j} \\ \beta_{i-1,j} = e_{i+1,j} + L_{i-1,j}e_{2j} - L_{i-2,j}(e_{3j} + L_{1j}e_{2j}) \\ \qquad + q_{i+1,d} - L_{i-2,j}q_{3d} - \dfrac{\partial \alpha_{ij}}{\partial e_{2j}}(e_{3j} + L_{1j}e_{2j} + q_{3d}) \\ \qquad - \displaystyle\sum_{l=3}^{i-1} \frac{\partial \alpha_{ij}}{\partial e_{lj}}(e_{l+1,j} + L_{l-1,j}e_{2j} - L_{l-2,j}(e_{3j} \\ \qquad + L_{1j}e_{2j}) + q_{l+1,d} - L_{l-2,j}q_{3d})(3 \le i \le n-1) \\ \beta_{n-1,j} = -\dfrac{\partial \alpha_{nj}}{\partial e_{2j}}(e_{3j} + L_{1j}e_{2j} + q_{3d}) \\ \qquad - \displaystyle\sum_{l=3}^{n-1} \frac{\partial \alpha_{nj}}{\partial e_{lj}}(e_{l+1,j} + L_{l-1,j}e_{2j} - L_{l-2,j}(e_{3j} \\ \qquad + L_{1j}e_{2j}) + q_{l+1,d} - L_{l-2,j}q_{3d}) \end{cases} \tag{23}$$

Based on the proposed observer (16), using the ideas in the state feedback, we have the following results.

Theorem 2: For system (1), under Assumption 1, control laws

$$u_{1j} = -k_1 z_{1j} - \sum_{i \in \mathcal{N}_j} a_{ji}(z_{1j} - z_{1i}) + u_{1d} \tag{24}$$

$$\begin{aligned} u_{2j} = & -k_n z_{nj} - \sum_{i \in \mathcal{N}_j} a_{ji}(z_{nj} - z_{ni}) + L_{n-2,j}(\widehat{\xi}_{1j} \\ & +L_{1j}q_{2j})u_{1j} + u_{2d} - L_{n-2,j}q_{3d}u_{1d} \\ & +\sum_{l=0}^{n-1} \frac{\partial \alpha_{n,j}}{\partial u_{1d}^{[l]}} u_{1d}^{[l+1]} + \frac{\partial \alpha_{nj}}{\partial e_{2j}}((e_{3j} + L_{1j}e_{2j})u_{1d} \\ & +\tilde{\xi}_{1j}u_{1j}) + \sum_{l=3}^{n-1} \frac{\partial \alpha_{n,j}}{\partial e_{lj}}(e_{l+1,j} + L_{l-1,j}e_{2j} \\ & +L_{l-2,j}(e_{3j} + L_{1j}e_{2j}))u_{1d} - \beta_{n-1,j}, \\ & (1 \le j \le m) \end{aligned} \tag{25}$$

make (3)-(6) satisfied, where the control parameters are chosen as in Lemmas 2-3.

Control laws (24)-(25) solve the output feedback formation control problem. In the control laws, we only use the output states. The convergence rate of the observer errors can be adjusted by the control parameters L_{ij}. The formation errors and the tracking errors are dependent on the control parameters $[k_1, \ldots, k_n]$ and a_{ji}, the observer parameter L_{ij}, and the communication topology.

V. SIMULATION

This section presents a simulation validation of performance.

Assume there are five car-like mobile robots which are indexed by 1, 2, 3, 4, and 5. The kinematic model of robot j is:

$$\begin{cases} \dot{x}_j = R_j v_{1j} \cos\theta_j, \dot{y}_j = R_j v_{1j} \sin\theta_j, \\ \dot{\theta}_j = R_j v_{1j} \tan\phi_j / l_j, \dot{\phi}_j = v_{2j} \end{cases} \tag{26}$$

where (x_j, y_j) represents the Cartesian coordinates of the middle point of the rear wheel axle, θ_j is the orientation of the robot body with respect to the X-axis, ϕ_j is the steering angle, l_j is the distance between the front and rear wheel-axle centers, R_j is the radius of rear driving wheel, v_{1j} is the angular velocity of the driving wheel, and v_{2j} is the steering velocity of the front wheels. With the state transformation

$$q_{1j} = x_j, q_{2j} = y_j, q_{3j} = \tan\theta_j, q_{4j} = \frac{\tan\phi_j}{l_j \cos^3\theta_j} \tag{27}$$

and the input transformation

$$\begin{cases} u_{1j} &= v_{1j} R_j \cos\theta_j \\ u_{2j} &= \dfrac{v_{2j} l_j \cos^2\theta_j + 3\sin\theta_j \sin^2\phi_j u_{1j}}{l_j^2 \cos^5\theta_j \cos^2\phi_j}, \end{cases} \tag{28}$$

system (26) is transformed into

$$\dot{q}_{1j} = u_{1j}, \dot{q}_{2j} = q_{3j} u_{1j}, \dot{q}_{3j} = q_{4j} u_{1j}, \dot{q}_{4j} = u_{2j} \tag{29}$$

which is a special case of (1). Obviously, states (q_{1j}, q_{2j}) is the position of robot j in the X-Y plane. States q_{3j} and q_{4j} are generalized angles related to the j-th robot. It should be noted that the transformation is local, i.e. $\theta_j \neq 0$ and $\phi_j \neq 0$.

Assume the desired formation is defined by $(h_{1x}, h_{1y}) = (2, 2)$, $(h_{2x}, h_{2y}) = (0, 4)$, $(h_{3x}, h_{3y}) = (-2, 2)$, $(h_{4x}, h_{4y}) = (-2, -2)$, $(h_{5x}, h_{5y}) = (2, -2)$ (See Fig 1). The desired trajectory of the center of the group of mobile robots is a line and is generated by a virtual robot with states: $x_d = t$, $y_d = t$, $\theta_d = 1$, $\phi_d = 0$. So $q_{1d} = x_d = t$, $q_{2d} = y_d = t$, $q_{3d} = \tan\theta_d = 1, q_{4d} = 0$.

Assume the digraph $\mathcal{G}$ is fixed and denoted as $\mathcal{G}_1$. The neighbors of each robot are as follows: $\mathcal{N}_1 = \{3, 4\}$, $\mathcal{N}_2 = \{1, 4\}$, $\mathcal{N}_3 = \{2, 5\}$, $\mathcal{N}_4 = \{3, 5\}$, $\mathcal{N}_5 = \{1, 2\}$. Using controllers (10)-(12), Fig. 2 shows path of each robot. Fig. 3 shows logarithms of the norm of the formation error and the norm of the tracking error of the center of the group of mobile robots. From the figures, it can be seen that (3)-(6) are satisfied.

If the communication digraph $\mathcal{G}$ does not strongly connected, control laws (10)-(12) can still make (3)-(6) satisfied. For example, if the communication graph is: $\mathcal{N}_1 = \emptyset$, $\mathcal{N}_2 = \{1, 4\}$, $\mathcal{N}_3 = \{2, 5\}$, $\mathcal{N}_4 = \{3, 5\}$, $\mathcal{N}_5 = \{1, 2\}$. Fig. 4 shows path of each robot. Fig. 5 shows logarithms of the norm of the formation error and the norm of the tracking error of the center of the group of mobile robots. The simulation results demonstrate that (3)-(6) are still satisfied.

For output feedback formation control with communication graph $\mathcal{G}_1$, we apply the control laws (24)-(25) with the same control parameter values as before and $L_{1j} = 2$, $L_{2j} = 1$. Fig. 6 shows path of each robot. Fig. 7 shows logarithms of the norm of the formation error and the norm of the tracking error. The results in Fig 6 and Fig. 7 show

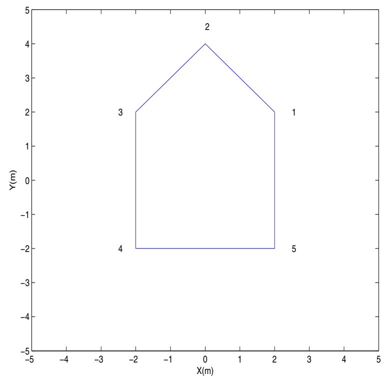

Fig 1. Desired formation.

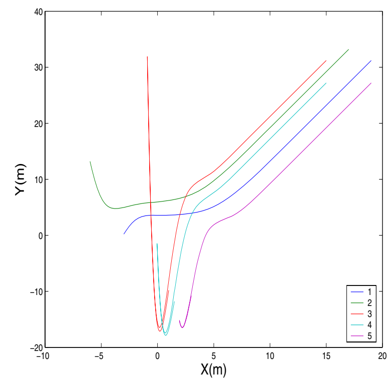

Fig. 2. Path of each robot (strong communication interconnection).

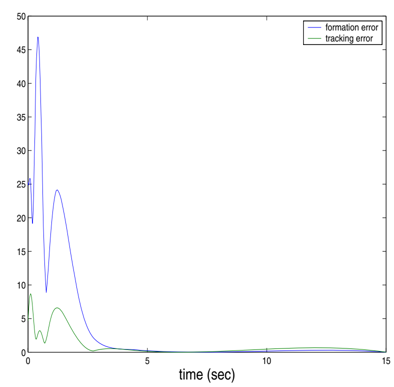

Fig. 3. Logarithms of the norm of the formation error and the norm of the tracking error (strong communication interconnection).

that the output control law (24)-(25) also make the group of robots form the desired formation.

VI. Conclusion

We consider the formation control of nonholonomic mobile robots. State feedback and output feedback controllers are proposed, which render the robot team to a given formation moving along a desired trajectory. Simulation results demonstrate satisfactory performances.

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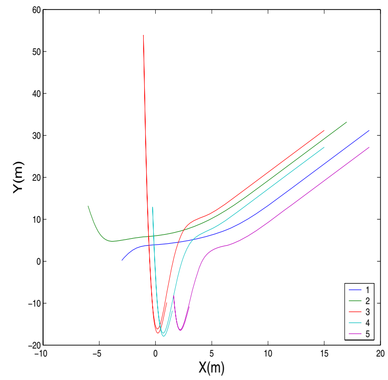

Fig. 4. Path of each robot (weak communication interconnection).

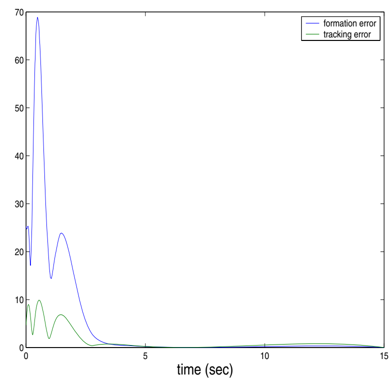

Fig. 5. Logarithms of the norm of the formation error and the norm of the tracking error (weak communication interconnection).

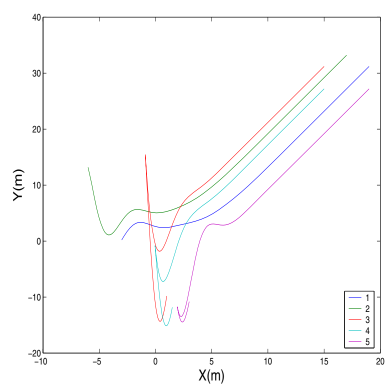

Fig 6. Path of each robot (output feedback)

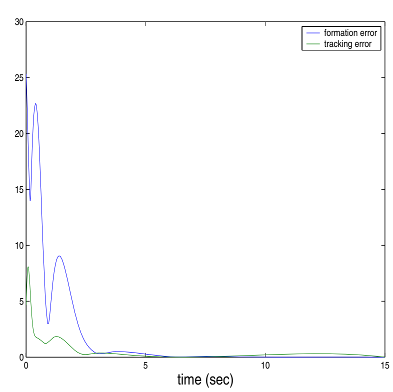

Fig 7. Logarithms of the norm of the formation error and the norm of the tracking error (output feedback)