Proceedings of the 2006 IEEE
International Conference on Mechatronics and Automation
June 25 - 28, 2006, Luoyang, China

Nonlinear Enhancement of Weak Signals Using Optimization Theory *

Xingxing Wu, Zhong-Ping Jiang, Daniel W. Repperger, and Yi Guo

***Abstract*— Stochastic resonance (SR) is a phenomenon that performance of the nonlinear system can be improved with the addition of optimal amount of noise. Stochastic resonance has been increasingly used for signal processing. The output of the nonlinear bistable dynamic system with white Gaussian noise input can be used to restore the weak input signal, if the similarity between the input signal and the output can be maximized. This paper will first use the optimization theory to show that the normalized power norm describing the similarity will reach a larger maximum when tuning both system parameters and noise intensity, compared with that of only adjusting noise intensity (classical stochastic resonance) or only adjusting system parameters. Then, computer simulations are performed to verify this proposal and demonstrate its application in signal processing.**

***Index Terms*— Optimization, Stochastic Resonance, Signal Processing**

I. INTRODUCTION

Noise is usually thought to be annoying and should be removed from the system. In some nonlinear systems, however, the addition of some extra amount of noise has been shown to be helpful. This phenomenon is called Stochastic resonance (SR)[1][2], and only exists in certain nonlinear systems. For these systems, the synchronization between the input signal and the noise will happen when the noise intensity is adjusted properly. In these cases, the system performance, such as the output signal-to-noise ratio or mutual information, will benefit from the noise. The improvement of the system performance can be maximized if the noise intensity is adjusted to an optimal level. This phenomenon was first revealed by Benzi in 1981 to explain the periodically recurrent ice ages [3]. Since then, stochastic resonance has been continuously attracting considerable attention of researchers. Basically, the stochastic resonance involves four elements: nonlinear system, information-carrying input signal, noise, and performance measure[8]. Many kinds of nonlinear systems have been shown to yield stochastic resonance phenomenon, such as the static systems[4] and dynamic systems[1]. The input signal can be periodic[1] or aperiodic[5]. The classical stochastic resonance requires the input signal to be subthreshold signal[1]. Recently, it was found that the input signals can be arbitrary and are not limited to weak signals. This is characterized by the suprathreshold stochastic resonance[6]. Not only the white Gaussian noise[1], but also the colored[9] and non-Gaussian noise[10], can generate a stochastic resonance effect. In order to describe the stochastic resonance more exactly, many quantifiers have been proposed as the performance measures, such as signal-to-noise ratio[1], power norm[5], and mutual information[7]. Stochastic resonance has found applications in many different areas, such as noise enhanced tactile sensation[11], the application of suprathreshold stochastic resonance to cochlear implant coding[12]. Another important application is in signal processing. It has been used for signal detection[13], signal transmission[15], signal estimation[17], and image processing[8]. The detector based on stochastic resonance can improve the robustness of the detector and its performance can compare with the locally optimum detectors(LOD)[14]. When the information is transmitted through a large parallel summing array, the noise can enhance performance up to approximately half the theoretical noiseless channel capacity[16]. The Bayesian estimator using stochastic resonance technique will achieve the minimum of the mean square estimation error when estimating the frequency of a periodic signal corrupted by a phase noise[17]. In order to make the noise useful, the stochastic resonance effect should be realized. For the traditional stochastic resonance, the stochastic resonance is realized by adjusting noise intensity[1]. Recently, the parameter-tuning stochastic resonance shows that tuning system parameters is a better method to realize stochastic resonance in some situations, especially when the initial input noise intensity is already beyond the resonance region[18][19][20]. The chosen performance measure will reach a higher/lower maximum/minimum, compared with that by adjusting noise intensity. This paper will apply optimization theory to show that the maximal normalized power norm of the bistable double-well system can be further increased by tuning system parameters and noise intensity at the same time, compared with that of parameter-tuning stochastic resonance and that of

*This work has been partially supported by the Polytechnic CATT Center sponsored by New York State, NSF grants ECS-009317, OISE-0408925, and DMS-0504462, and an Air Force contract

Xingxing Wu and Zhong-Ping Jiang are with the Department of Electrical and Computer Engineering, Polytechnic University, Brooklyn, NY 11201 USA xwu03@utopia.poly.edu, zjiang@control.poly.edu

Daniel. W. Repperger is with the Air Force Research Laboratory, AFRL/HECP, Wright-Patterson AFB, OH 45433 USA Daniel.Repperger@wpafb.af.mil

Yi Guo is with the Department of Electrical and Computer Engineering, Stevens Institute of Technology, Hoboken, NJ 07030, USA yguo1@stevens.edu

classical stochastic resonance. The normalized power norm is the performance measure describing the similarity between the input signal and the output of this nonlinear system with white Gaussian noise input. The increase of the similarity between the input and the output by this scheme will benefit the restoration of the weak input signal and has potential applications in signal processing.

The rest of the paper is organized as follows. Section II will describe the nonlinear bistable system and the related performance measure. In Section III, we will show it is possible to further enhance the stochastic resonance effect with tuning system parameters and noise intensity by working with a more general nonlinear bistable system. Section IV will focus on the simulations and the application of this scheme in signal processing. Finally, Section V concludes the paper.

II. Nonlinear bistable double-well stochastic resonance system

The nonlinear bistable double-well system can be expressed in the following equation[5]

$$\frac{dx}{dt} = -\frac{\partial U}{\partial x} + \xi(t), \tag{1}$$

where $U(x)$ is the potential function.

The symmetric potential function with a fluctuating barrier is given by[5]

$$U(x) = -[A - S(t)]\frac{x^2}{2} + \frac{x^4}{4}, \tag{2}$$

where A is positive and is taken as a tuning parameter in this paper. $S(t)$ is the input signal with zero-mean average. $\xi(t)$ is white Gaussian noise with zero mean and autocorrelation of $\langle \xi(t)\xi(s)\rangle = 2D\delta(t-s)$. The angular brackets denote the ensemble average.

In order to demonstrate the stochastic resonance effect, the cross-correlation measures (power norm C_0, and normalized power norm C_1) are taken as the performance measures

$$C_0 = max\{\overline{S(t)R(t+\tau)}\}, \tag{3}$$

$$C_1 = \frac{C_0}{[\,\overline{S^2(t)}\,]^{1/2}\{\,\overline{[R(t) - \overline{R(t)}\,]^2}\,\}^{1/2}}, \tag{4}$$

where $R(t)$ is used as the system response characterized by mean transition rate of the system. The overbar denotes an average over time. τ is a time lag.

The cross-correlation describes the similarity between the input signal and the system output. Usually, it is hard to find an explicit expression for the above cross-correlation. If the input signal is both weak, i.e, $\overline{S(t)^2} \ll A^2$, and also of Gaussian-distribution, the ensemble averaged power norm $\langle C_0\rangle$ and the ensemble averaged normalized power norm $\langle C_1\rangle$ can be approximated by[5]

$$\langle C_0\rangle \simeq Q_0\Delta_0 \exp[-\Theta_0 + \Delta_0^2\,\overline{S^2(t)}/2]\,\overline{S^2(t)}, \tag{5}$$

$$\langle C_1\rangle \simeq \frac{\Delta_0[\,\overline{S^2(t)}\,]^{1/2}}{(e^{(\Delta_0^2\,\overline{S^2(t)}\,)} - 1 + \sigma(D)Q_0^{-2}e^{(2\Theta_0 - \Delta_0^2\,\overline{S^2(t)}\,)})^{1/2}}, \tag{6}$$

where:

$$\sigma(D) = K_1\langle\overline{R(t)}\rangle, \quad \langle\overline{R(t)}\rangle \simeq Q_0 \exp[\,-\Theta_0 + \Delta_0^2\,\overline{S^2(t)/2}\,],$$
$$Q_0 = K_0 A/\sqrt{2}\pi, \quad \Theta_0 = A^2/4D, \quad \Delta_0 = A/2D.$$

$\langle R(t)\rangle$ is the ensemble-averaged escape rate. Also, the angular brackets denote an ensemble average.

It was revealed in [5] that this performance measure or the similarity between input and output will be maximized when an optimal amount of additional noise is added into the system. This is termed aperiodic stochastic resonance (ASR). Now, we will discuss the method in which the similarity can be further enhanced so that the input signal can be better restored from noise.

III. Enhancement of stochastic resonance effect

Usually, the stochastic resonance effect is realized either by adjusting noise intensity (classical stochastic resonance) or by tuning system parameters (parameter-tuning stochastic resonance), but not both. In some cases, the parameter-tuning method is better than the classic method. Intuitively, the stochastic resonance effect can be further enhanced if both the system parameters and the noise intensity are adjusted at the same time. This, however, is not always true, even if the performance measure is affected by both the system parameters and the noise intensity. In [21], the bit error rate (BER) will be minimized if the noise intensity is not adjusted and fixed at the initial level, while the system parameter is tuned to the optimal value. In our recent paper[22], we demonstrate that, for the bistable double-well dynamic system with Gaussian-distributed input signal and fluctuating barrier, it is possible to further increase the normalized power norm $\langle C_1\rangle$ by tuning system parameter and noise intensity at the same time.

In [22], we require that two additional parameters should be introduced into the system, in order to make this mechanism possible for different weak input signals. These two new system parameters in [22], however, do not have direct physical meaning. This makes the mechanism of tuning system parameters and noise intensity a little hard to understand. Now, we will change to a new system model in which these system parameters will have direct physical meaning. Based on this model, we will also prove that this mechanism is true not only for the Gaussian-distributed input signals, but also for the general weak input signals.

The new potential function of this nonlinear system is modified as

$$U(x) = -[A - S(t)]\frac{x^2}{2} + \frac{x^4}{4X_b^2}, \tag{7}$$

where X_b is one of the two new system parameters.

By introducing another new system parameter τ_a, the nonlinear dynamic system is now described by the following equation

$$\tau_a \dot{x}(t) = [A - S(t)]x(t) - \frac{x^3(t)}{X_b^2} + \xi(t), \quad (8)$$

where the definitions of $S(t)$ and $\xi(t)$ are same as (1).

In this nonlinear system, the system parameters are τ_a, X_b and A. Parameter τ_a will affect the system response time and parameter X_b will affect the barrier height of the potential function of this system. Parameter A is used to shift the input signal. All these three system parameters have direct physical meaning and their influences on the the system performance measure $\langle C_1 \rangle$ are easier to understand, compared with the one used in [22].

From [5], we know the ensemble-averaged escape rate can be expressed as follow, if the parameter $\tau_a = 1$

$$\langle R(t) \rangle \simeq \frac{1}{2\pi}\sqrt{U''(x_{min})|U''(x_{max})|}e^{(\frac{U(x_{min})-U(x_{max})}{D})}, \quad (9)$$

where U is the potential function, x_{min} is one of the local minimizers and x_{max} is the local maximizer.

The new nonlinear system equation and the new potential function will affect the $\langle R(t) \rangle$. The method to derive the approximation of $\langle C_1 \rangle$ is similar to the one to derive (6). Also, for the general weak input signal case, we can derive the final approximation of normalized power norm $\langle C_1 \rangle$, if the condition $\Delta^2 \overline{S(t)^2} \ll 1$ is met

$$\langle C_1 \rangle \simeq \frac{\Delta s}{(\Delta^2 s^2 + \frac{\Delta^4 s^4}{2} + \frac{K_1}{Q}(1 - \frac{\Delta^2 s^2}{2} + \frac{\Delta^4 s^4}{8})e^{c\tau_a Q\Delta})^{1/2}}, (10)$$

where:
$Q = \frac{K_0 A}{\sqrt{2}\tau_a \pi}$, $\Theta = \frac{\tau_a X_b^2 A^2}{4D} = c\tau_a \Delta Q$, $\Delta = \frac{\tau_a X_b^2 A}{2D}$, $s = \sqrt{\overline{S(t)^2}}$.

In order to investigate whether the stochastic resonance effect of the above nonlinear system can be further enhanced by tuning system parameters and noise intensity at the same time, we need to check whether the following constrained optimization problem has global maximizer

$$max\langle C_1 \rangle, \quad (11)$$
$$\text{subject to:} \quad A > 0,\ s^2 \ll A^2,\ \Delta^2 s^2 \ll 1,\ D_0 \le D \le D_1$$

The constrain of $\overline{S(t)^2} \ll A^2$ comes from the requirement on weak input signal. Parameter A is positive and $[\,A - \overline{S(t)}\,]$ is also positive. In addition, $\Delta^2 s^2 \ll 1$ should be satisfied in order to make (10) valid. According to [5], the theoretic expression $\langle C_1 \rangle$ can still predict its real shape, even if the noise intensity is beyond the range of its validity. Also, we assume here that the noise cannot be removed. So, the only requirements on the noise intensity are that it cannot be less than its initial value D_0, and it can not be arbitrarily large.

For this optimization problem, we will take parameters A and D as the optimal parameters, while parameter τ_a and X_b will be taken as the supporting parameters which are used to ensure the optimization problem (11) has solution, as shown later. In order to simplify the calculation, the direct optimal parameters of (11) are Δ and Q, which are in turn the functions of A and D.

We now prove that (11) has one and only one global maximizer. Here, we will first define the corresponding unconstrained optimization problem as

$$max\langle C_1 \rangle. \quad (12)$$

Proposition 1: The unconstrained optimization problem (12) has one and only one pair of parameters (Q^*, Δ^*) satisfying the first-order necessary condition for a local maximizer.

Proof: According to the first-order necessary condition of this optimization problem (12), we have

$$\frac{\partial \langle C_1 \rangle}{\partial \Delta} = 0 \quad \text{and} \quad \frac{\partial \langle C_1 \rangle}{\partial Q} = 0. \quad (13)$$

Then, we can derive $c\tau_a \Delta Q = 1$, and

$$-s^4\Delta^3 + c\tau_a e K_1(1 + s^2\Delta^2/2 - 3s^4\Delta^4/8) = 0. \quad (14)$$

Let

$$f(\Delta) = -s^4\Delta^3 + c\tau_a e K_1(1 + s^2\Delta^2/2 - 3s^4\Delta^4/8). \quad (15)$$

Obviously, there is at least one solution (Q^*, Δ^*) satisfying this first-order necessary condition, because

$$f(0) = c\tau_a e K_1 > 0 \quad \text{and} \quad f(+\infty) = -\infty. \quad (16)$$

Let

$$f_1(\Delta) = -s^4\Delta^3, \quad (17)$$
$$f_2(\Delta) = c\tau_a e K_1(1 + s^2\Delta^2/2 - 3s^4\Delta^4/8). \quad (18)$$

Function $f_1(\Delta)$ is a monotonically decreasing function. Function $f_2(\Delta)$ will first increase with Δ and then decrease to $-\infty$. From these facts, we can prove that the first-order necessary condition can only have one positive solution.

■

Proposition 2: The system parameter τ_a can continuously adjust (Q^*, Δ^*) satisfying the first-order necessary condition. ,

Proof: The system parameter τ_a affects $f_2(\Delta)$, but not $f_1(\Delta)$. From the special characteristics of these two functions, we can find out that the increase of τ_a will also increase the value of Δ^* satisfying (14). If τ_a is getting close to zero, Δ^* will also approach zero. From this, we complete the proof of this proposition. ■

Proposition 3: The unconstrained optimization problem (12) has one and only one local maximizer when the input is small and the system parameters τ_a and X_b are chosen properly.

Proof: From Proposition 1, we know the first-order necessary condition only has one solution (Q^*, Δ^*). We now prove this solution will also satisfy the second-order sufficient condition for a local maximizer, that is the Hessian matrix is negative definite at the point (Q^*, Δ^*).

According to Proposition 2, the system parameter τ_a can be adjusted properly so that the requirement $s^2\Delta^{*2} \ll 1$ can be satisfied. From this, we can get

$$-c\tau_a eK_1 s^2\Delta^{*2} + s^2\Delta^*(-4+2s^2\Delta^{*2}) \\ -c\tau_a eK_1 s^2\Delta^{*2}(2-3s^2\Delta^{*2}/4) < 0, \tag{19}$$

and

$$-s^4\Delta^4/8 + (-1+s^2\Delta^{*2}/2) < 0. \tag{20}$$

From (14), (19), and (20), it follows that $\frac{\partial^2\langle C_1\rangle}{\partial\Delta^2}$, $\frac{\partial^2\langle C_1\rangle}{\partial Q^2}$, and $\frac{\partial^2\langle C_1\rangle}{\partial\Delta\partial Q}$ are all negative at $\Delta = \Delta^*$, and $Q = Q^*$.

The Hessian matrix is defined as

$$\begin{pmatrix} \frac{\partial^2\langle C_1\rangle}{\partial\Delta^2} & \frac{\partial^2\langle C_1\rangle}{\partial\Delta\partial Q} \\ \frac{\partial^2\langle C_1\rangle}{\partial Q\partial\Delta} & \frac{\partial^2\langle C_1\rangle}{\partial Q^2} \end{pmatrix}$$

To prove the Hessian matrix is negative definite, we need to verify its determinant value is positive.

At $\Delta = \Delta^*$ and $Q = Q^*$, the numerator of this Hessian matrix determinant value can be simplified as:

$$s^4\Delta^{*3}(2-2s^2\Delta^{*2}) + c\tau_a eK_1 s^4\Delta^{*4}(\frac{14}{8} - \frac{9s^2\Delta^{*2}}{8}) \\ +(s^4\Delta^{*3} + \frac{3c\tau_a eK_1 s^4\Delta^{*4}}{8} - \frac{c\tau_a eK_1 s^2\Delta^{*2}}{2}) \\ +\frac{s^8\Delta^{*7}}{2} + \frac{15c\tau_a eK_1 s^8\Delta^{*8}}{64}. \tag{21}$$

Its numerator will be positive, if $s^2\Delta^{*2} \ll 1$. Also, its denominator is positive. From the standard test on negative-definiteness of a symmetric matrix, it follows the Hessian matrix is negative definite. This completes the proof of Proposition 3. ■

Proposition 4: The constrained optimization problem (11) with small input has one and only one global maximizer, if the system parameters τ_a and X_b are chosen properly.

Proof: According to Proposition 3, (12) has one and only one local maximizer (Q^*, Δ^*). Also, from Proposition 2, Δ^* can be continuously adjusted by the system parameter τ_a such that $s^2\Delta^{*2} \ll 1$. In this case, the requirement for $s^2 \ll A^{*2}$ will also be satisfied and system parameter A^* will be positive, because of $A^* = 2/\Delta^*$.

The constraints on the noise intensity can also be satisfied by tuning system parameters τ_a and X_b, because of $D^* = \tau_a X_b^2/\Delta^{*2}$. So, the constrained optimization problem (11) has one and only one local maximizer.

It is obvious that the only local maximizer is also the global maximizer of (11). This completes the proof of this proposition. ■

Proposition 4 reveals that the results of [22] can be extended to the more general weak input signal case, when the nonlinear system model with more physical meaning is adopted. This system's maximal normalized power norm $\langle C_1\rangle$ can be further increased with this scheme, compared with that of either tuning system parameters or adjusting noise intensity.

IV. Enhancement of Weak Signals

The traditional method to restore the weak signal corrupted by noise will try to remove noise from the signal. The method based on stochastic resonance (SR), on the contrary, can improve the performance measure, such as signal-to-noise ratio, with the addition of an extra amount of noise. A critical task in developing the SR-based signal processor is how to realize stochastic resonance and how to enhance stochastic resonance effect. In this paper, the output of the bistable double-well dynamic system described by (8) is corrupted by the white Gaussian noise. A SR-based method will be used to restore the weak input signal from the system output. The normalized power norm $\langle C_1\rangle$ is adopted as the performance measure. Obviously, a larger $\langle C_1\rangle$ value means that the input signal and the system output are more similar, or the system output carries more information about the input signal. This, in turn, means that the input signal can be better restored from the output of this nonlinear system. In this case, the mechanism to further improve the maximal $\langle C_1\rangle$ by tuning system parameters and nose intensity will have practical usage. It can better restore the signal from noise than the traditional SR methods.

In order to demonstrate this mechanism's better enhancement of weak signal compared with the traditional stochastic resonance methods, simulations are performed. The first simulation is to directly compare the maximal $\langle C_1\rangle$ reached by three different methods: (1) adjusting system parameters and noise at the same time; (2) only adjusting system parameters; (3) only adjusting noise intensity. The simulation result is shown in Fig. 1. From this figure, it is obvious that the mechanism proposed in this papers gives the best performance, especially for the weak input signal case.

Now, we will directly deal with the system output $x(t)$ and compare their waveforms by changing system parameter values and noise intensity.

Fig. 2 is the simulation model. In this model, A, a, and b are the system parameters and $a = 1/\tau_a, b = 1/(\tau_a X_b^2)$. The noise intensity D will affect the output of the White Noise block. The Constant block with value "shift" is used to shift the average value of the input pulse to zero. The User-Defined Functions block is used to generate x^3.

Fig. 3 shows some of the system outputs under different system parameter values and different noise intensity for the same input signal with an amplitude of 0.005. From this simulation, it is easy to notice that the similarity between input and output, or the input signal information carried by the system output, is greatly affected by the choices of the system parameters and noise intensity. It will be maximized for the properly chosen values. This can be even more

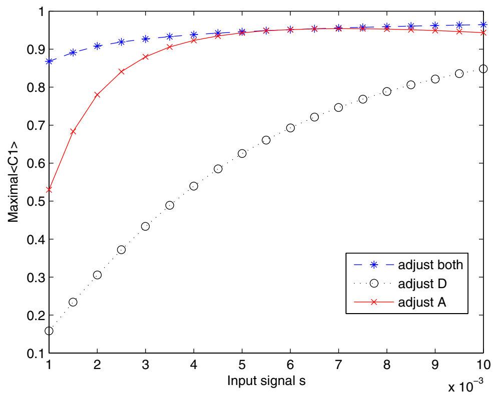

Fig. 1. Comparison of Maximal $\langle C_1 \rangle$

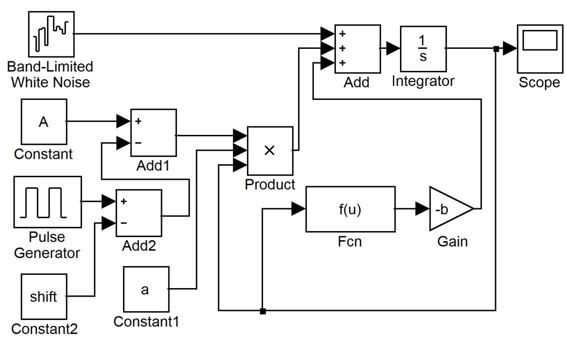

Fig. 2. Simulation Model

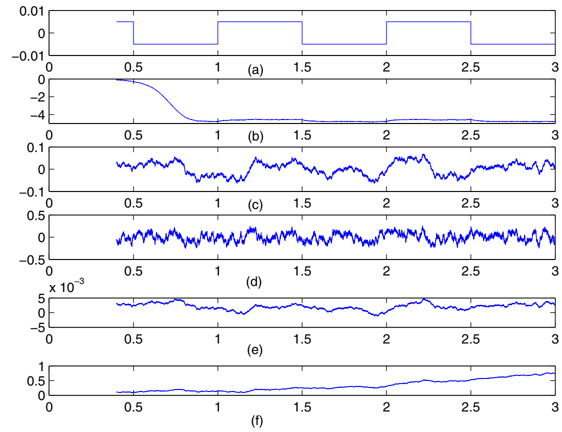

Fig. 3. System Output for Different Parameter Values (a) original input signal (b) A=0.11, a=100, b=0.5, D=0.01 (c) A=1, a=0.08, b=3600, D=0.01 (d) A=1, a=0.08, b=3600, D=1 (e) A=1, a=0.08 b=3600, D=0.00001 (f) A=1, a=1, b=1, D=0.01

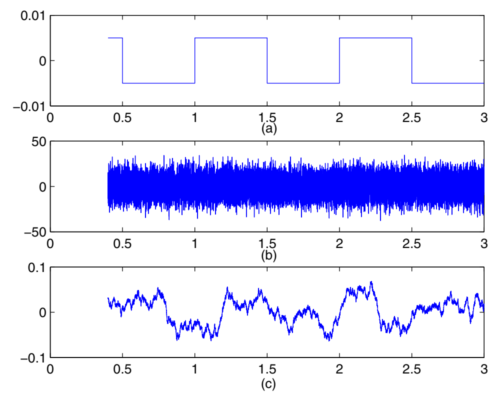

Fig. 4. System Output for noisy input (a) original input signal (b) noisy input signal (c) system output at A=0.5, a=0.08, b=3600, D=0.01

obvious, if combined with other signal processing methods. For the analog signals, the method introduced in [20] might be adopted. For the binary signals, we might decide the binary value using the detection methods.

This SR-based method with tuning system parameters and noise intensity is useful when this nonlinear SR dynamic system is a part of the whole system under investigation. In this case, tuning the system parameters and noise intensity to maximize the enhancement of the weak input signal will benefit the rest of the system. For example, it will be easier to process an input with higher signal-to-noise ratio.

In reality, the input signal $S(t)$ and the noise $\xi(t)$ are usually mixed together. The $S(t)$ and $\xi(t)$ in the system model (8) is in fact $S(t) + \xi(t)$. In this case, the bistable double-well dynamic system is acted as a nonlinear filter. Its performance will also be affected by the choices of system parameter values and noise intensity. Fig. 4 demonstrates the system output under the noisy input signal. The overall features of this new system need further investigation.

V. Conclusion

This paper first reveals that the results of [22] can be extended to a modified bistable double-well nonlinear dynamic system with more general weak input signals. In this nonlinear system, the system parameters have more physical meaning and the tuning of these system parameters is easier to understand. Then, the mechanism of tuning system parameters and noise intensity at the same time is applied to help the recovery of weak signal from the noisy output of the bistable double-well dynamic system. The method based on stochastic resonance plays a unique role in the nonlinear enhancement of weak signals corrupted by noise, compared with traditional denoising filters. Our future work will be directed at extending this mechanism to other SR-based filters and comparing the performance with other filters in different applications.

REFERENCES

[1] L. Gammaitoni, P. Hänggi, P. Jung and F. Marchesoni, "Stochastic resonance," *Reviews of Modern Physics*, vol. 70, no. 1, pp. 223-287, 1998.

[2] T. Wellens, V. Shatokhin and A. Buchleitner, "Stochastic resonance," *Reports on Progress in Physics*, 67, pp. 45-105, 2004.

[3] R. Benzi, A. Sutera and A. Vulpiani, "The mechanism of stochastic resonance," *J. Phys. A*, vol. 14, no. 11, L453, 1981.

[4] F. Chapeau-Blondeau and X. Godivier, "Theory of stochastic resonance in signal transmission by static nonlinear systems," *Phys. Rev. E*, vol. 55, no. 2, 1997.

[5] J. J. Collins, Carson C. Chow, Ann C. Capela and Thomas T. Imhoff, "Aperiodic stochastic resonance," *Physical Review E*, vol. 54, no. 5, 1996.

[6] N. G. Stocks, "Suprathreshold stochastic resonance in multilevel threshold systems," *Physical Review Letters*, vol. 84, no. 11, 2000.

[7] G. Deco and B. Schrmann, "Stochastic resonance in the mutual information between input and output spike trains of noisy central neurons," *Phys. D*, vol. 117, pp. 276-282, 1998.

[8] F. Chapeau-Blondeau, "Stochastic resonance and the benefit of noise in nonlinear systems," *Noise, Oscillators and Algebraic Randomness - From Noise in Communication Systems to Number Theory*, pp. 137-155; M. Planat, ed., Lecture Notes in Physics, Springer (Berlin), 2000.

[9] D. Nozaki, D. J. Mar, P. Grigg, and J. J. Collins, "Effects of colored noise on stochastic resonance in sensory neurons," *Physical Review Letters*, vol. 82, no. 11, 1999.

[10] B. Kosko, and Sanya Mitaim, "Robust stochastic resonance: signal detection and adaption in impulsive noise," *Physical Review E*,vol. 64, 051110, 2001.

[11] J. J. Collins, T. T. Imhoff and P. Grigg "Noise-enhanced tactile sensation," *Nature*,383(6603):770, 1996.

[12] N. G. Stocks and D. Allingham "The application of suprathreshold stochastic resonance to cochlear implant coding," *Fluctuaton and Noise Letters*, vol. 2, pp. 169-181, 2002.

[13] A. A. Saha, G. V. Anand, "Design of detectors based on stochastic resonance," *Signal Processing*, vol. 83, issue 6, pp. 1193-1212, 2003.

[14] S. Zozor and P. -O. Amblard, "On the use of stochastic resonance in sine detection," *Signal Processing*, vol. 82, no. 3, pp: 353-367, 2002.

[15] X. Godivier and F. Chapeau-Blondeau, "Noise-assisted signal transmission in a nonlinear electronic comparator: experiment and theory," *Signal Processing* 56: 293-303, 1997.

[16] N. G. Stocks, "Information transmission in parallel threshold arrays: Suprathreshold Stochastic Resonance," *Physical Review E*, vol. 63, 2001.

[17] F. Chapeau-Blondeau and D. Rousseau, "Noise-enhanced performance for an optimal Bayesian estimator," *IEEE Transactions on Signal Processing*, vol. 52, no. 5, May 2004.

[18] B. Xu, F. Duan and F. Chapeau-Blondeau, "Comparison of aperiodic stochastic resonance in a bistable system realized by adding noise and by tuning system parameters," *Physical Review E* 69, 2004.

[19] B. Xu, F. Duan, R. Bao and J. Li, "Stochastic resonance with tuning system parameters: the application of bistable systems in signal processing," *Chaos, Solitons and Fractals*, 13:633-644, 2002.

[20] B. Xu, J. Li and J. Zheng, "How to tune the system parameters to realize stochastic resonance," *J. Phys. A: Math. Gen.*, 36 ,11969-11980, 2003.

[21] B. Xu, F. Duan and F. Chapeau-Blondeau, "Comparison of aperiodic stochastic resonance in a bistable system realized by adding noise and by tuning system parameters," *Physical Review E* 69, 2004.

[22] X. Wu, Z. Jiang and D. W. Repperger, "Enhancement of stochastic resonance by tuning system parameters and adding noise simultaneously," accepted by *Amercian Control Conference*, 2006.

[23] J. Nocedal and S. J. Wright, *Numerical Optimization*, Springer-Verlag New York, Inc., 1999.