Proceedings of the ASME 2010 Dynamic Systems and Control Conference
DSCC2010
September 12-15, 2010, Cambridge, Massachusetts, USA

DSCC2010-4193

ROBUST CONSENSUS OUTPUT TRACKING OF MULTI-AGENT SYSTEMS WITH DIRECTED COMMUNICATIONS

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ABSTRACT

In this paper, we study the consensus output tracking control for multi-agent systems with high-order dynamics under directed communication topology. Time-varying reference is assumed to be available to a subgroup of a team. A leader-follower scheme is applied and robust consensus control is developed so that the reference is treated as disturbances to those agents with no access to the reference. The control scheme avoids estimation of the derivatives of neighbor's states through measurement as done in previous work and guarantees a finite L_2-gain from the reference to an transformed output. Simulation results show satisfactory performances.

1 INTRODUCTION

Recently, increasing attention has been paid to coordinated control of multi-agent systems due to its numerous potential applications in space-based interferometers, combat, surveillance, reconnaissance systems, hazardous material handling, and distributed reconfigurable sensor networks. Consensus, to achieve an agreement on certain quantities of interest, is a critical problem in coordinated control of multiple agents. Based on eigenvalue analysis, the consensus problem was studied in [6, 7, 17, 20–22, 24, 26, 28, 31]. The passivity-based framework in [1] provides an explicit way for finding Lyapunov functions on undirected communication graphs. In [4] [5], the authors study an output synchronization condition for balanced communication topologies based on passivity. In [15], a model transformation is used to transform the original system into a reduced-order system so that a sufficient condition can be obtained for all agents to reach consensus with a desired H_∞ performance.

These results are based on the fact that the consensus equilibrium is a weighted average or a weighted power mean of the initial conditions of all agents' states. Thus, all agents achieve consensus to some unknown constant. However, there are applications that need all agents to achieve a desired common reference, which may be time-varying. To track a time-varying consensus reference, most existing consensus algorithms rely on the assumption that all agents know the time-varying group reference.

Consensus with a constant reference is studied in [11] with undirected switching inter-vehicle communications, and in [12, 16] under a directed fixed interaction topology. Consensus algorithm with a time-varying reference is proposed in [9] with a variable undirected interaction topology. In [23, 25], taking consensus reference as a virtual leader, consensus tracking algorithms are proposed to track a time-varying consensus reference with a directed topology. In these work, the estimate of the neighbors' velocity, which is obtained by calculating numerical differentiation of the local neighbors states, is needed by each agent to achieve consensus tracking. The dynamics of agents considered in these work are single-integrator or double-integrator. In [30], consensus problem of multi-agent systems with higher-order dynamics is studied. However, the same linear model applies to each agent. In [27], the authors study lth-order ($l \leq 3$) consensus algorithms, present the idea of higher-order consensus with a leader, and introduce the concept of an lth-order model-reference consensus problem.

We consider the consensus output tracking under more general conditions: the dynamics of agents are modeled as higher-order linear systems, and they may be different among agents. Moreover, we assume that time-varying tracking reference is available to a subgroup of a team that has a spanning tree communication topology. A reduced-order transformation is introduced to transform the consensus output tracking problem into a finite L_2-gain control problem, which is then solved using robust control techniques. Comparing to the existing work on consensus output tracking, our proposed method can achieve consensus tracking of a time-varying reference for agents that are modeled as higher-order dynamics. Also, we achieve it by using robust consensus control techniques without estimation of the derivatives of neighbors' states through measurement as done in [25]. A performance index in terms of an L_2-gain is guaranteed from the reference to the transformed output.

The subsequent sections are organized as follows: Section 2 introduces the related graph theory and Preliminaries provides the statement of Consensus Output Tracking problem. In section 3, the main results are given on consensus output tracking. Section 4 shows the simulation results. In section 5, we present the conclusion.

2 Preliminaries and Problem Formulation

2.1 Graph Theory in Consensus

A digraph $\mathcal{G}$ consists of a triple $(\mathcal{V},\mathcal{E},\mathcal{A})$, where $\mathcal{V}$ is a finite nonempty set of nodes, $\mathcal{E}\in\mathcal{V}^2$ is a set of ordered pairs of nodes defined as edges, and a weighted adjacency matrix $\mathcal{A}=[a_{ij}]$ with nonnegative adjacency elements a_{ij}. The node indexes belong to a finite index set $I=\{1,2,...,n\}$. An edge of $\mathcal{G}$ is denoted by $e_{ij}=(v_i,v_j)$, with the weight a_{ij}, that is $e_{ij}\in\mathcal{E}\Longleftrightarrow a_{ij}>0$, and we assume $a_{ii}=0$ and $a_{ij}=1, i\neq j$ for all $i,j\in I$, in unspecified. The set of neighbors of node v_{ij} is denoted by $N_i=\{v_i\in\mathcal{V}:(v_i,v_j)\in\mathcal{E}\}$.

A directed path in a digraph is a sequence of edges as $(v_{i_1},v_{i_2}),(v_{i_2},v_{i_3}),\ldots,(v_{i_m},v_{i_{m+1}})$, where $v_{i_j}\in\mathcal{V}$ and $e_{i_ji_{j+1}}\in\mathcal{E}, j=1,\ldots,m$. A directed graph has a directed spanning tree if there exists at least one node that all the other node could reach it following directed path directions.

The *graph Laplacian* associated with the graph $\mathcal{G}$ is defined as

$$\mathcal{L}(\mathcal{G})=L=\Delta-\mathcal{A} \tag{1}$$

The diagonal matrix $\Delta=[\Delta_{ij}]$ where $\Delta_{ij}=0$ for all $i\neq j$ and $\Delta_{ii}=\deg_{\text{out}}(v_i)$. Since every row sum is zero, the Laplacian matrix always has a zero eigenvalue with the right eigenvector of one. We denote as

$$\lambda_1=0,\ \ w_r=\mathbf{1}=(1,1,\ldots,1)^T \tag{2}$$

Lemma 1. *If a digraph $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$ has a spanning tree and with a Laplacian matrix L, there exists a non-singular matrix M such that $L=M^{-1}JM$, where J is the Jordan block with $J=\operatorname{diag}\{J_1,0\}$ where $-J_1$ is a $(n-1)\times(n-1)$ Hurwitz matrix.*

Proof. Since the digraph $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$ has a spanning tree, one of eigenvalues of L is zero and others are greater than zero. The Jordan Canonical Form Theorem ([18]) guarantees that there exists a non-singular matrix M such that $L=M^{-1}JM$, where J has the form $J=\operatorname{diag}\{J_1,0\}$.

2.2 Finite L_2-Gain

Definition 1. *([29]) Let $G:L_{2e}\to L_{2e}$, where L_{2e} is the extended L_2 space. Then G is said to have finite L_2 gain if there exist finite constants γ_2 and b_2 such that for all $T\geq 0$*

$$\int_0^T \|G(u)\|^2\,dt<\gamma_2^2\int_0^T\|u\|^2\,dt+b_2,\quad \forall u\in L_{2e}. \tag{3}$$

G is said to have finite L_2 gain with zero bias if b_2 in (3) is equal to zero.

2.3 Problem Formulation

Let $x_i\in R^p$ be the state of the ith agent. We consider that the dynamics of agents have the general form:

$$\begin{aligned}\dot{x}_i&=A_ix_i+b_iu_i\\ y_i&=c_ix_i,\end{aligned} \tag{4}$$

where the system matrix A_i is of dimension $p\times p$, the input matrix $b_i\in R^p$, and the output matrix c_i is of $1\times p$. The transfer function from the input u to the output y is

$$H_i(s)=c_i(s\mathbf{I}-A_i)^{-1}b_i:=\frac{P_i(s)}{Q_i(s)} \tag{5}$$

We make the following assumptions:

Assumption 1. *The systems (5), $i=1,\ldots,n$, have a uniform relative degree: r, which is defined by $r=\deg Q_i(s)-\deg P_i(s)$ ([8]).*

Assumption 2. *The system matrices A_i, $i=1,\ldots,n$, are stable, i.e., the eigenvalues of A_i are located on the left half of the complex plane.*

Assumption 3. *Suppose that the consensus reference, denoted by ξ_d, satisfies*

$$\begin{aligned}\dot{\xi}_d&=f_0(t,\xi_d)\\ y_r&=\xi_d\end{aligned} \tag{6}$$

where $f_0(\cdot,\cdot)$ is $r-1$ times differentiable and $\frac{d^{r-1}f_0}{dt}(t,\xi_d(t))$ is bounded.

We have the following two different assumptions on the communication graph:

Assumption 4. *The communication graph contains a spanning tree, and the reference is available to all agents.*

Assumption 5. *The communication graph contains a spanning tree, and the reference is available to at least one root agent of the spanning tree.*

Under Assumptions 1-3 and 4 or 1-3 and 5, the Consensus Output Tracking problem is to design distributed control laws, $u_i(x_i, x_j)$, $j \in N_i$, $i = 1, \ldots, n$, such that the outputs y_i of all agents converge to the time-varying reference y_r.

3 Main Results

By Assumption 1, we have ([10])

$$\begin{array}{ll} c_i A_i^k b_i = 0 & k = 0, \ldots, r-2 \\ c_i A_i^{r-1} b_i = s_i \neq 0. & \end{array} \tag{7}$$

Taking derivative of y_i r times with respect to t gives

$$\begin{aligned} y_i^{(r)} &= c_i A_i^r x_i + c_i A_i^{r-1} b_i u_i \\ &= c_i A_i^r x_i + s_i u_i. \end{aligned} \tag{8}$$

The decentralized state feedback controllers

$$u_i = \frac{1}{s_i} \left(-c_i A_i^r x_i + v_i \right), \tag{9}$$

reduces the input-output map into the following linear form

$$y_i^{(r)} = v_i. \tag{10}$$

3.1 Adding Consensus Tracking Reference as a Virtual Leader

We introduce the consensus reference (6) as a virtual leader with the output y_r. We name the virtual leader as the $(n+1)$th agent without loss of generality. Prim's algorithm ([3, 19]) or Kruskal's algorithm ([14]) can be applied to find all root agents to a spanning tree. By Assumption 4 or 5, the $(n+1)$th agent is the root agent of the expanded spanning tree. Thus, the Laplacian matrix $\hat{L} = \left[\hat{l}_{ij}\right]$ corresponding to the new graph is a $(n+1) \times (n+1)$ matrix with $\hat{l}_{i(n+1)} = 0$, $i = 1, \ldots, n$.

Denoting $y_{n+1} = y_r = \xi_d$, the dynamics

$$y_{n+1}^{(r)} = v_{n+1} \tag{11}$$

is the same as the dynamics of tracking reference (6) when $v_{n+1} = f_0^{(r-1)}(t, \xi_d)$.Note that if the tracking reference is a constant, $f_0(t, \xi_d)$ in (6) is zero. Thus, v_{n+1} is zero.

Let $y = [y_1, \ldots, y_{n+1}]^T \in R^{n+1}$. The dynamics of the output y of all $n+1$ agents is

$$y^{(r)} = v, \tag{12}$$

where $v = [v_1, \ldots, v_{n+1}]^T$.

3.2 Transformation to Stabilization Problem

From Lemma 1, there exists a non-singular matrix $\hat{M} \in R^{n+1}$ such that $\hat{L} = \hat{M}^{-1} \hat{J} \hat{M}$, where $\hat{J}$ is the Jordan form with $\hat{J} = \mathrm{diag}\{\hat{J}_1, 0\}$ where $-\hat{J}_1$ is a $n \times n$ Hurwitz matrix.

From the definition of $\hat{J}$, it is easy to see that

$$\hat{J} = \begin{bmatrix} I_n \\ \mathbf{0}_{1\times n} \end{bmatrix} \hat{J}_1 \begin{bmatrix} I_n & \mathbf{0}_{n\times 1} \end{bmatrix}. \tag{13}$$

Thus, the Lapalace matrix $\hat{L}$ can be represented by

$$\hat{L} = \hat{M}^{-1} \begin{bmatrix} I_n \\ \mathbf{0}_{1\times n} \end{bmatrix} \hat{J}_1 \begin{bmatrix} I_n & \mathbf{0}_{n\times 1} \end{bmatrix} \hat{M}. \tag{14}$$

We employ a dimension-reduced transformation

$$z_{n\times 1} = \begin{bmatrix} I_n & \mathbf{0}_{n\times 1} \end{bmatrix} \hat{M} y. \tag{15}$$

Differentiating it r times with respect to time gives

$$z^{(r)} = \begin{bmatrix} I_n & \mathbf{0}_{n\times 1} \end{bmatrix} \hat{M} y^{(r)}. \tag{16}$$

Substituting (12) into (16) yields

$$z^{(r)} = \begin{bmatrix} I_n & \mathbf{0}_{n\times 1} \end{bmatrix} \hat{M} v. \tag{17}$$

Let

$$v = W \hat{M}^{-1} \begin{bmatrix} I_n \\ \mathbf{0}_{1\times n} \end{bmatrix} \hat{J}_1 \hat{v} + \hat{M}^{-1} \begin{bmatrix} \mathbf{0}_{n\times 1} \\ 1 \end{bmatrix} \frac{1}{d} f_0^{(r-1)}, \tag{18}$$

where d is the $n+1$th element of the vector $\hat{M}^{-1} \begin{bmatrix} \mathbf{0}_{n\times 1} \\ 1 \end{bmatrix}$ and $W = \mathrm{diag}\{w_1, \ldots, w_{n+1}\}$ with $w_i > 0$, $i = 1, \ldots, n+1$. We obtain that

$$z^{(r)} = B \hat{v}, \tag{19}$$

where $B = \hat{J}_1 \begin{bmatrix} I_n & \mathbf{0}_{n\times 1} \end{bmatrix} \hat{M} W \hat{M}^{-1} \begin{bmatrix} I_n \\ \mathbf{0}_{1\times n} \end{bmatrix}$.

In the case when W is an identical matrix, the input matrix B in the system (19) is $\hat{J}_1$. Note that the second term in (18) picks a particular element in the null space of the one-dimension-reduced transformation (15) in order to have $v_{n+1} = f_0^{(r-1)}$. To solve the consensus output tracking problem, a distributed control is required. Thus, the control $\hat{v}$ in the system (19) has the following form:

$$\hat{v} = -\sum_{l=1}^{r} k_l z^{(l-1)}. \tag{20}$$

To verify it, we substitute (20) into the first term in (18), which gives

$$\begin{aligned} & W\hat{M}^{-1}\begin{bmatrix} I_n \\ \mathbf{0}_{1\times n} \end{bmatrix} \hat{J}_1 \hat{v} \\ &= -W\hat{M}^{-1}\begin{bmatrix} I_n \\ \mathbf{0}_{1\times n} \end{bmatrix} \hat{J}_1 \sum_{l=1}^{r} k_l z^{(l-1)} \\ &= -\sum_{l=1}^{r} k_l W\hat{M}^{-1}\begin{bmatrix} I_n \\ \mathbf{0}_{1\times n} \end{bmatrix} \hat{J}_1 \begin{bmatrix} I_n & \mathbf{0}_{n\times 1} \end{bmatrix} \hat{M}y \\ &= -\sum_{l=1}^{r} k_l W\hat{L}y. \end{aligned} \tag{21}$$

Since W is diagonal, from (21), we can see that the consensus controller uses the neighbors' information only. Also, from (21), W takes a role of weighting coefficients of each agent's distributed controllers.

Since $\hat{L}$ has a decomposition (14) and from the definition of z, we have $\hat{L}y = \hat{M}^{-1}\begin{bmatrix} I_n \\ \mathbf{0}_{1\times n} \end{bmatrix} \hat{J}_1 z$. Based on this relationship, we can conclude that $z = 0$ yields $\hat{L}y = 0$. That is, the consensus output is achieved. Thus, the consensus output is transformed into a stability problem for the system (19) using control (20).

3.3 Consensus Output Tracking with All Agents Having Access to $f_0^{(r-1)}$

Firstly, we solve the consensus output tracking problem defined in Section 2.3 under Assumptions 1-3 and 4. We assume that all agents can access the reference. Thus, the information $f_0^{(r-1)}$ is available to all agents.

For convenience, we rearrange the coordinate in a compact form $Z = [z_1^T, \ldots, \left(z^{(r-1)}\right)^T]^T \in R^{r\times n}$, then the system (19) can be written in a compact form:

$$\dot{Z} = A_Z Z + B_Z v \tag{22}$$

where $A_Z = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ldots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix}$ and $B_Z = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \vdots \\ B \end{bmatrix}$ From the definition of B, it is easy to see that $\operatorname{rank}(B) = n$. Thus, the pair (A_Z, B_Z) is controllable.

Theorem 1. *Under Assumptions 1-3 and 4, if there exist scalars k_i $i = 1, \ldots, r$, and a $rn \times rn$ symmetric positive definite matrix P such that*

$$\Theta < 0, \tag{23}$$

where

$$\begin{aligned} \Theta = {} & P\left(A_Z - B_Z([k_1, \ldots, k_r] \otimes \mathbf{I}_n)\right) \\ & + \left(A_Z - B_Z([k_1, \ldots, k_r] \otimes \mathbf{I}_n)\right)^T P, \end{aligned}$$

and the notation $\Theta < 0$ represents that the symmetric matrix Θ is negative definite,

then the distributed controller

$$u_i = \frac{1}{s_i}\left(-C_i A_i^r x_i + d_i f_0^{(r-1)} - \sum_{l=1}^{r} k_l w_i \sum_{j\in N_i} \hat{l}_{ij}\right) \tag{24}$$

where d_i is the ith element of the vector $\hat{M}^{-1}\begin{bmatrix} \mathbf{0}_{n\times 1} \\ 1 \end{bmatrix}$, solves the consensus output tracking problem.

Proof. Choose a Lyapunov function candidate as follows:

$$V(t) = Z^T(t) P Z(t). \tag{25}$$

Taking time derivative of $V(t)$ along with the solutions of (22) with controllers $v = ([k_1, \ldots, k_r] \otimes \mathbf{I}_n) Z$ gives

$$\begin{aligned} \dot{V}(t) &= 2Z^T(t) P \left(A_Z - B_Z([k_1, \ldots, k_r] \otimes \mathbf{I}_n)\right) Z(t) \\ &= Z^T(t) \Theta Z(t), \end{aligned} \tag{26}$$

where $\Theta = P\left(A_Z - B_Z([k_1, \ldots, k_r] \otimes \mathbf{I}_n)\right) + \left(A_Z - B_Z([k_1, \ldots, k_r] \otimes \mathbf{I}_n)\right)^T P$. By the assumption that $\Theta < 0$, we have

$$\dot{V}(t) \le \lambda_{\max}(\Theta) \|Z(t)\|^2, \tag{27}$$

where $\lambda(\Theta)$ is the largest eigenvalue of Θ. Since $\Theta < 0$, $\lambda(\Theta)$. From (27), the system (22) is exponentially stable by Lyapunov stability theorem ([13]).

Based on the statement in the last paragraph in Section 3.2, from the stability of (22) we can conclude that consensus output tracking problem is solved.

3.4 Consensus Output Tracking with $f_0^{(r-1)}$ Known by Partial Agents

In the previous section, the consensus output tracking problem is solved under an assumption that all agents can access the reference. Though it achieves consensus output tracking asymptotically, it is not practical. It is reasonable that only some agents can access to the reference. In this section, we solve the consensus output tracking problem under Assumption 5. The information $f_0^{(r-1)}$ is only available to at least one of the root agents. Thus, for other agents that cannot access this information, $f_0^{(r-1)}$ is treated as disturbances.

Defined a $(n+1)\times(n+1)$ matrix $H=\operatorname{diag}\{h_i\}$ as $h_i=1$ if $\hat{L}(i,(n+1))\neq 0$, $h_i=0$ if $\hat{L}(i,(n+1))=0$ and $h_{n+1}=1$. To keep controller distributed, we apply the following controller

$$v=-\sum_{l-1}^{r}k_l W\hat{L}y+D\hat{M}^{-1}\begin{bmatrix}\mathbf{0}_{n\times 1}\\ 1\end{bmatrix}\xi^{(r)}(t). \tag{28}$$

Substituting the controller (28) and $u_{n+1}=\xi_d^{(r)}$ into (16) and considering the relationship $H=I-(I-H)$ gives

$$z^{(r)}=Bv+P_\omega\omega \tag{29}$$

where $\omega=f_0^{(r-1)}$, $v=-\sum_{i-1}^{r}k_i z^{(r-1)}$ and $P_\omega\in R^n=\begin{bmatrix}I_n & \mathbf{0}_{n\times 1}\end{bmatrix}\hat{M}(\mathbf{I}_{n+1}-H)\hat{M}^{-1}[\mathbf{0}_{1\times n},1]$. Comparing it to the system (19), a disturbance term appears.

We rearrange the coordinate in a compact form $Z=[z_1^T,\ldots,\left(z^{(r-1)}\right)^T]^T\in R^{r\times n}$, (29) can be written into:

$$\dot{Z}=A_Z Z+B_Z u+P_Z\omega \tag{30}$$

where $P_Z=\left[\mathbf{0}^T,\mathbf{0}^T,\cdots,P_\omega^T\right]^T$.

We focus on the state $z=\left[\mathbf{I}_n,\mathbf{0}_{n\times(r-1)n}\right]Z$ to see if the mapping from the disturbance ω to the controlled output $z(t)$ has finite L_2-gain $\gamma>0$ or equivalently, the closed-loop system satisfies the following dissipation inequality

$$\int_0^T\|z(t)\|^2dt<\gamma^2\int_0^T\|\omega(t)\|^2dt,\quad\forall\omega\in L_{2e},\quad\forall T>0.$$

Theorem 2. *Under Assumptions 1-3 and 5, if there exist scalars γ, k_i $i=1,\ldots,r$, and a symmetric positive definite matrix P_Z satisfying*

$$\begin{bmatrix}\Theta+C_Z^TC_Z & PP_Z\\ P_Z^TP & -\gamma^2\end{bmatrix}<0 \tag{31}$$

where

$$\begin{aligned}\Theta=&P\left(A_Z-B_Z([k_1,\ldots,k_r]\otimes\mathbf{I}_n)\right)\\&+\left(A_Z-B_Z([k_1,\ldots,k_r]\otimes\mathbf{I}_n)\right)^TP\end{aligned},$$

then the distributed controller

$$u_i=\frac{1}{s_i}\left(-C_iA_i^rx_i+h_id_if_0^{(r-1)}-\sum_{l=1}^{r}k_lw_i\sum_{j\in N_i}\hat{l}_{ij}\right) \tag{32}$$

which reaches the each agent's neighbor information only, solves the consensus output tracking problem and the following L_2-gain performance inequality holds

$$\int_0^T\left\|\hat{L}y\right\|^2dt<\lambda_{max}^2\left(\hat{J}_1\right)\gamma^2\int_0^T\|\omega(t)\|^2dt,\quad\forall\omega\in L_{2e}.$$

Proof. Choose a Lyapunov function candidate as follows:

$$V(t)=Z^T(t)PZ(t). \tag{33}$$

Firstly, we proved the stability of the system (30 with $\omega=0$ in the proof of Theorem 1. Now, we discuss the performance of the system (30) with disturbance $\omega(t)$.

Taking derivative of $V(t)$ along with the solutions of (30) with controllers $v=([k_1,\ldots,k_r]\otimes\mathbf{I}_n)Z$ with respect to t gives

$$\begin{aligned}\dot{V}(t)&=2Z^T(t)P\left(A_Z-B_Z([k_1,\ldots,k_r]\otimes\mathbf{I}_n)\right)Z(t)\\&=Z^T(t)\Theta Z(t)-\gamma^2\left\|\omega-\tfrac{1}{\gamma^2}P_Z^TPZ(t)\right\|_2^2\\&\quad+\tfrac{1}{\gamma^2}PP_ZP_Z^TP+\gamma^2\|\omega\|_2^2.\end{aligned} \tag{34}$$

By Schur Complement Formula [2], (31) is equivalent to

$$\Theta+C_Z^TC_Z+\gamma^{-2}PP_ZP_Z^TP<0. \tag{35}$$

Substituting (35) into (34) yields

$$\begin{aligned}\dot{V}(t)&\le\gamma^2\|w\|_2^2-\|z\|_2^2-\gamma^2\left\|\omega-\tfrac{1}{\gamma^2}P_Z^TPZ(t)\right\|_2^2\\&\le\gamma^2\|w\|_2^2-\|z\|_2^2.\end{aligned} \tag{36}$$

Note that the left-hand side of (36) is the derivative of V along the trajectories of the system (30). Integrating (36) yields

$$\begin{aligned}&2V(Z(\tau))-2V(Z(0))\\&\quad\le\gamma^2\textstyle\int_0^\tau\|w\|_2^2dt-\int_0^\tau\|z\|_2^2dt,\end{aligned} \tag{37}$$

where $Z(t)$ is the solution of (30) for a given $\omega \in \mathcal{L}_2[0,\infty)$. Using $V(Z) \geq 0$, we obtain

$$\int_0^\tau \|z\|_2^2 dt \leq \gamma^2 \int_0^\tau \|w\|_2^2 dt + 2V(Z_0), \tag{38}$$

which is equivalent to the mapping from ω to z has finite L_2-gain γ.

From the definition of z, we have $\hat{L}y = \begin{bmatrix} I_n & \mathbf{0}_{n\times 1} \end{bmatrix} \hat{J}_1 z$. Thus, we have $\|\hat{L}y\|_2 \leq \lambda_{max}(\hat{J}_1)\gamma\|\omega\|_2$.

4 Simulation

We consider a group of agents with 3rd order linear dynamics:

$$\begin{aligned} \dot{x}_i &= Ax_i + Bu_i, \quad i = 1,2,3,4,5 \\ y_i &= Cx_i, \end{aligned} \tag{39}$$

where $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$.

We consider that the communication topology contains more than one possible spanning trees shown in Figure 1. The corresponding Laplacian matrix is

$$L = \begin{bmatrix} 2 & -1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{40}$$

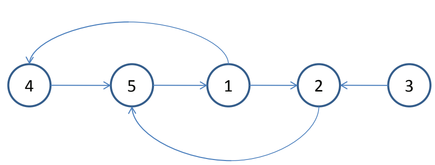

Figure 1. Communication Topology with Spanning Trees

We assume that the desired output tracking trajectory is $\xi_d(t) = sin(t)$. Figure 1 shows us that there are 2 possible spanning tree with different leaders, $3 \rightarrow 2 \rightarrow 5 \rightarrow 1 \rightarrow 4$ with 4 as a leader, and $4 \rightarrow 5 \rightarrow 1 \rightarrow 2$, $3 \rightarrow 2$ with 2 as a leader. We assume that both agents 4 and 2 are able to access the reference as shown in Figure 2. The Laplacian Matrix $\hat{L}$ of the new digraph with the virtual leader is

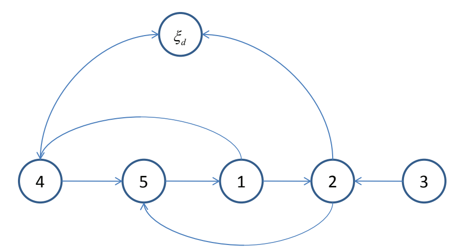

Figure 2. The Communication Topology after Adding a Virtual Leader

$$\hat{L} = \begin{bmatrix} 2 & -1 & 0 & -1 & 0 & 0 \\ 0 & 2 & 0 & 0 & -1 & -1 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & -1 & -1 \\ -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{41}$$

From Lemma 1, there exists a non-singular matrix $\hat{M}$ such that $\hat{L} = \hat{M}^{-1} \begin{bmatrix} I_n \\ \mathbf{0}_{1\times n} \end{bmatrix} \hat{J}_1 \begin{bmatrix} I_n & \mathbf{0}_{n\times 1} \end{bmatrix} \hat{M}$ where

$$\hat{J}_1 = \begin{bmatrix} 2.3478 & 1.0289 & 0 & 0 & 0 \\ -1.0289 & 2.3478 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0.3044 \end{bmatrix} \tag{42}$$

We apply Theorem 2 to calculate controller as follows:

$$u = -k_1\hat{L}y - k_2\hat{L}\dot{y} - k_3\hat{L}\ddot{y} + \begin{bmatrix} 0 \\ -\sin(t) - k_1(x_2(1) - \sin(t)) \\ -k_2(x_2(2) - \cos(t)) - k_3(x_2(3) + \sin(t)) \\ 0 \\ -\sin(t) - k_1(x_4(1) - \sin(t)) \\ -k_2(x_4(2) - \cos(t)) - k_3(x_4(3) + \sin(t)) \\ 0 \end{bmatrix}$$

which guarantees that the outputs of all agents achieve consensus and follow the desired trajectory $\sin(t)$, which is verified in Fig. 3 and 4. If $\dddot{\xi}_d(t)$ is available to Agent 4 only, the Laplacian Matrix $\hat{L}$ of the new digraph with the virtual leader is

$$\hat{L} = \begin{bmatrix} 2 & -1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & -1 & -1 \\ -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{43}$$

From Theorem 2, the distributed control is

$$
\begin{aligned}
u = & -k_1\hat{L}y - k_2\hat{L}\dot{y} - k_3\hat{L}\ddot{y} \\
& + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\sin(t) - k_1(x_4(1) - \sin(t)) \\ -k_2(x_4(2) - \cos(t)) - k_3(x_4(3) + \sin(t) \\ 0 \end{bmatrix}
\end{aligned}
$$

which solves the consensus output tracking problem. Fig. 5 and 6 show that the performance is a little different from the case when $\dddot{\xi}_d(t)$ is available to both Agent 2 and 4. We can see that when the reference is available to more agents, the performance of consensus tracking is better.

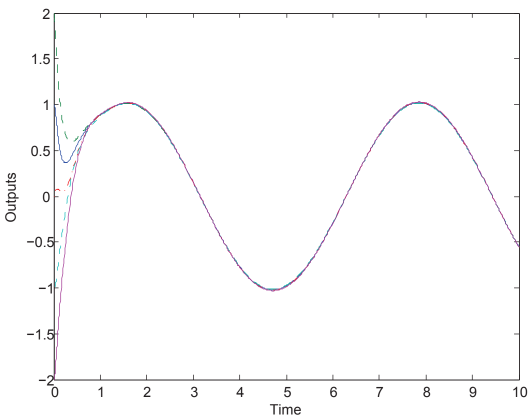

Figure 3. Consensus Output Tracking with $\dddot{\xi}_d$ available for Agents 2 and 4

5 Conclusions

In this paper, we study the consensus output tracking control of multi-agent systems with higher-order dynamics under directed communication topologies. A reduced-order transformation is found to transform the consensus problem to a stabilization problem. When the tracking trajectory is time-varying, the reference is treated as disturbances for those agents that cannot access this information and the consensus output tracking problem is then solved using robust control techniques. The performance of consensus output tracking is measured by an L_2-gain. Simulation shows the effectiveness of our proposed algorithm and that the performance can be improved by increasing the number of agents that can access the reference.

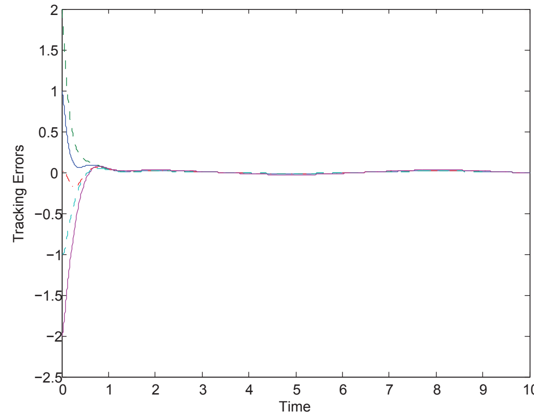

Figure 4. Tracking Errors with $\dddot{\xi}_d$ available for Agents 2 and 4

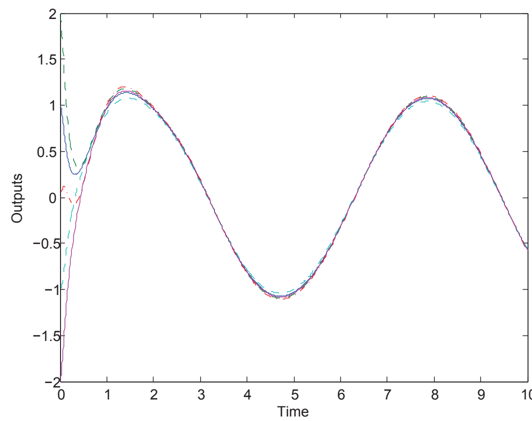

Figure 5. Consensus Output Tracking with $\dddot{\xi}_d$ available for Agent 4

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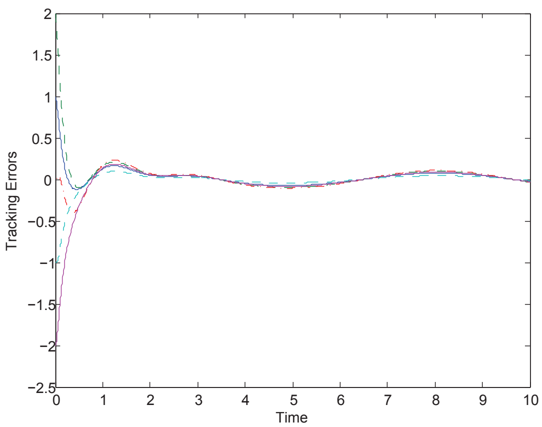

Figure 6. Tracking Errors with $\dddot{\xi}_d$ available for Agent 4

nonlinear systems with relative degree one. In *Recent Advances in Learning and Control, V.D. Blondel, S.P. Boyd, and H. Kimura, eds.*, pages 51–64. Springer Verlag, Berlin, 2008.

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